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Introduction

The general theme of the meeting concerns various aspects of interaction of complex variables and scientific computation, including related topics from function theory, operator theory (holomorphic function spaces), approximation theory and numerical analysis. Another important aspect of the CMFT meetings, previously held in Aveiro 2001, Joensuu 2005, Ankara 2009 and Shantou 2013, is to promote the creation and maintenance of contacts with scientists from diverse cultures. The Maria Curie-Skłodowska University (UMCS) in Lublin, Poland, will be hosting the eighth international conference on Computational Methods and Function Theory (CMFT 2017) to be held 10ľ15 July 2017. The CMFT 2017 meeting forms part of the official celebrations of the 700th anniversary of the foundation of the city of Lublin in 2017.

The international organizing committee of CMFT 2017

Ilpo Laine (University of Eastern Finland, FINLAND),

Doron Lubinsky (Georgia Institute of Technology, USA),

Maria T. Nowak (Maria Curie-Skłodowska University, POLAND),

Dariusz Partyka (John Paul II Catholic University of Lublin, POLAND),

Stephan Ruscheweyh (Würzburg University, GERMANY),

Edward B. Saff (Vanderbilt University, USA).

Plenary speakers

Daoud Bshouty, Technion, Israel Institute of Technology, ISRAEL,

Janne Heittokangas, University of Eastern Finland, Joensuu, FINLAND,

Aimo Hinkkanen, University of Illinois at Urbana-Champaign, USA,

Arno Kuijlaars, Katholieke Universiteit Leuven, BELGIUM,

Nir Lev, Bar-Ilan University, ISRAEL,

Norman Levenberg, Indiana University, Bloomington, USA,

Miodrag Mateljevic, University of Belgrade, SERBIA,

Kristian Seip, Norwegian University of Science and Technology, NORWAY,

Barry Simon, California Institute of Technology, USA,

Vilmos Totik, Szeged University and University of South Florida, HUNGARY/USA.

The CMFT 2017 conference is partially supported by the Ministry of Science and Higher Education in Poland, project no 624/1/P-DUN/2017.

General program

The Opening Ceremony, all Plenary Lectures and the Closing Ceremony will take place at the building D (Institute of Informatics), Lecture Hall 105

Monday, 10 July 2017

9:00	Opening Ceremony
9:15	CMFT Young Scientist Award Ceremony **Laudatio:** Doron Lubinsky
9:30-10:30	Prize Lecture **Nir Lev**, *Fourier quasicrystals*
10:30 - 11:00	Coffee Break
11:00 - 12:40	Parallel Sessions
13:00 - 14:30	Lunch
14:30 - 15:30	Plenary Lecture **Chair:** Janne Heittokangas **Daoud Bshouty**, *Harmonic mappings via real and complex analysis: thirty years after*
15:30 - 16:00	Coffee Break
16:00 - 18:15	Parallel Sessions
19:00	Reception

Tuesday, 11 July 2017

9:00 - 10:00	Plenary Lecture **Chair:** Matti Vuorinen **Kristian Seip**, *Analysis for Dirichlet series and the Riemann zeta function*
10:00 - 10:30	Coffee Break
10:30 - 12:45	Parallel Sessions
13:00 - 14:30	Lunch
14:30 - 15:30	Plenary Lecture **Chair:** Vladimir Andrievskii **Barry Simon**, *Szegö-Widom asymptotics for Chebyshev polynomials on subsets of* $\boldsymbol{R}$
15:30 - 16:00	Coffee Break
16:00 - 18:15	Parallel Sessions

Wednesday, 12 July 2017 - Excursion to Janowiec and Kazimierz Dolny

Thursday, 13 July 2017

9:00 - 10:00	Plenary Lecture **Chair:** Maria Nowak **Miodrag Mateljević**, *Interior estimates for elliptic PDE, quasiconformal and HQC mappings between Lyapunov Jordan domains*
10:00 - 10:30	Coffee Break
10:30 - 12:45	Parallel Sessions
13:00 - 14:30	Lunch
14:30 - 15:30	Plenary Lecture **Chair:** Lisa Lorentzen **Aimo Hinkkanen**, *Growth of Painlevé transcendents*
15:30 - 17:00	Coffee Break & Poster Session
17:00 - 17:30	Problems Section **Chair:** Vilmos Totik **Yuri Zelinskii**, *Some open problems in analysis*
17:30 - 18:30	Contributed open problems (with discussion)

Friday, 14 July 2017

9:00 - 10:00	Plenary Lecture **Chair:** Aimo Hinkkanen **Norman Levenberg**, *Multivariate approximation, convex bodies and pluripotential theory*
10:00 - 10:30	Coffee Break
10:30 - 12:45	Parallel Sessions
13:00 - 14:30	Lunch
14:30 - 15:30	Plenary Lecture **Chair:** Abdallah Lyzzaik **Janne Heittokangas**, *Exponential polynomials and oscillation theory*
15:30 - 16:00	Coffee Break
16:00 - 18:15	Parallel Sessions
19:30 - …	Conference Dinner

Saturday, 15 July 2017

9:00 - 10:00	Plenary Lecture **Chair:** Paul Gauthier **Arno Kuijlaars**, *Universality for conditional measures of the sine point process*
10:00 - 10:30	Coffee Break
10:30 - 11:30	Plenary Lecture **Chair:** Stephan Ruscheweyh **Vilmos Totik**, *Critical points of polynomials*
11:30	Closing Ceremony & Lunch

Schedule for Parallel Sessions

Monday, July 10

Session 1, Room 105, Building D

Time	Speaker	Title	Chair
11:00-11:30	Józef Zając	*The Schwartz range domain for harmonic mappings of the unit disc with boundary normalization*	Miodrag Mateljević
11:35-12:05	Dariusz Partyka	*Quasiconformality of harmonic mappings in the unit disk*	
12:10-12:40	Armen Grigoryan	*Extensions of harmonic functions of the complex plane slit along an interval*	
16:00-16:30	Matti Vuorinen	*Metric and quasiconformal mappings*	Dariusz Partyka
16:35-17:05	Parisa Hariri	*Hyperbolic type metrics in geometric function theory*	
17:10-17:40	Toshiyuki Sugawa	*A Möbius invariant and a nearly hyperbolic distance of a punctured sphere*	
17:45-18:15	Edmund Yik Man Chiang	*When does a formal finite-difference expansion become "real"?*	

Session 2, Room 156, Building C

Time	Speaker	Title	Chair
11:00-11:30	Kuldeep Singh Charak	*Some generalizations of the fundamental normality test*	Toshiyuki Sugawa
11:35-12:05	Tomoki Kawahira	*On dynamical and parametric Zalcman functions*	
12:10-12:40	Mondher Chouikhi	*Quadratic differentials related to Heun equations*	
16:00-16:30	Andrzej Sołtysiak	*Ideals with at most countable hull in certain algebras of analytic functions*	Kristian Seip
16:35-17:05	Marek Ptak	*Asymmetric truncated Toeplitz operators and conjugations*	
17:10-17:40	Kamila Kliś-Garlicka	*C-symmetric, skew-symmetric operators, and reflexivity*	
17:45-18:15	Piotr Dymek	*Multiplier algebras of weighted shifts on directed trees*	

Session 3, Room 217, Building D

Time	Speaker	Title	Chair
11:00-11:30	Alexander Dyachenko	*One helpful property of functions generating Pólya frequency sequences*	Dmitrii Karp
11:35-12:05	Luis Salinas	*On a geometric property of the Euler-Mascheroni sequence*	
12:10-12:40	Andrew Bakan	*Integral representation for the logarithm of the Riemann zeta function on the interval (0,1)*	
16:00-16:30	Oleksandr Vlasiuk	*Variable density node distribution: Riesz minimizers and irrational lattices*	Edward Saff
16:35-17:05	Franck Wielonsky	*Weighted Riesz potential theory and convergence of empirical measures*	
17:10-17:40	Vasudha Girgaonkar	*Coefficient bounds for two new subclasses of bi-univalent functions*	
17:45-18:15			

Session 4, Room 332, Building D

Time	Speaker	Title	Chair
11:00-11:30	Winfried Auzinger	*Adaptive integration of nonlinear evolution equations*	Janusz Godula
11:35-12:05	Turgay Bayraktar	*Universality principles for zeros of random polynomials*	
12:10-12:40			
16:00-16:30	Guilherme Silva	*S-contours for multiple orthogonal polynomials in the hermitian plus external source random matrix model*	Richard Fournier
16:35-17:05	Gunter Semmler	*Critical points of finite Blaschke products, Stieltjes polynomials and moment problems*	
17:10-17:40	Aaron Yeager	*Zeros of real random polynomials spanned by OPUC*	
17:45-18:15	Mikhail Tyaglov	*Non-real zeroes of homogeneous differential polynomials*	

Session 5, Room F VI, Building C

Time	Speaker	Title	Chair
16:00-16:30	Surendra Sunil Varma	*Region of variability for a subclass of analytic univalent functions*	Frode Rønning
16:35-17:05	Priyanka Sangal	*On a conjecture for trigonometric sums by S. Koumandos and S. Ruscheweyh*	
17:10-17:40	Thomas Rosy	*Logarithmic coefficients for certain subclasses of quasi-convex univalent functions*	
17:45-18:15	Prashant Batra	*Real entire functions, zero-location and structured minors with applications to the Riemann hypothesis and Turan's inequalities*	

Tuesday, July 11

Session 1, Room 105, Building D

Time	Speaker	Title	Chair
10:30-11:00	Eleanor Lingham	*Hayman's List - an update*	Barry Simon
11:05-11:35	Roger Barnard, Alex Solynin	*Applications of a "Two Point Boundary Variation"*	
11:40-12:10	Roger Barnard, Alex Solynin	*Applications of a "Two Point Boundary Variation"*	
12:15-12:45	Barbara Śmiarowska	*On the Julia-Carathéodory Theorem for functions with fixed initial coefficients*	
16:00-16:30	Yelda Aygar	*On the spectral analysis of a discrete Sturm–-Liouville equation*	Marek Ptak
16:35-17:05	Arthur Danielyan	*Interpolation by bounded analytic functions and related questions*	
17:10-17:40	Taneli Korhonen	*Zero sequences and factorization for weighted Bergman spaces*	
17:45-18:15	Wenjun Yuan	*Representations and applications of meromorphic solutions of some odd higher order algebraic differential equations*	

Session 2, Room 156, Building C

Time	Speaker	Title	Chair
10:30-11:00	Theodore Kilgore	*Constructive proof of the Weierstrass Theorem for weighted functions on unbounded intervals*	Edmund Y.-M. Chiang
11:05-11:35	Lenka Mihokovic	*Asymptotic expansions of some n-variable means*	
11:40-12:10	Roman Riser	*Universal subleading asymptotics of planar orthogonal polynomials*	
12:15-12:45	Brian Simanek	*Asymptotically optimal point configurations for Chebyshev constants*	
16:00-16:30	Abdallah Lyzzaik	*Planar harmonic mappings of bounded boundary rotation*	Daoud Bshouty
16:35-17:05	Michael Dorff	*Convolutions of univalent harmonic strip mappings*	
17:10-17:40	Andrzej Michalski	*Univalence criteria for local homeomorphisms with application to planar harmonic mappings*	
17:45-18:15	Muhammed Syam	*Numerical computation of fractional second-order Sturm-Liouville problems*	

Session 3, Room 217, Building D

Time	Speaker	Title	Chair
10:30-11:00	Fiana Jacobzon	*Holomorphic semicocycles in Banach spaces*	Hasi Wulan
11:05-11:35	Tomasz Łukasz Żynda	*Some properties of the Szegö kernel*	
11:40-12:10	Mark Elin	*Fixed points of holomorphic mappings*	
12:15-12:45	Zoltan Leka	*A Leibniz-type rule for random variables*	
16:00-16:30	Iulia-Roberta Bucur	*On some new results related to special functions*	Luis Salinas
16:35-17:05	Tomislav Buric	*New asymptotic expansions and improvements of approximation formulas for the gamma function*	
17:10-17:40	Dmitrii Karp	*Logarithmic convexity and concavity of generalized hypergeometric functions with respect to multiple parameter shifts*	
17:45-18:15	Elena Prilepkina	*Applications of integral representations of generalized hypergeometric functions*	

Session 4, Room 237, Building A

Time	Speaker	Title	Chair
10:30-11:00	Andriy Bandura	*Growth estimate of entire functions of bounded* ***L****-index in joint variables*	Alekos Vidras
11:05-11:35	Edyta Trybucka	*Extremal problems of some family of holomorphic functions of several complex variables*	
11:40-12:10	Paweł Wójcicki	*Bergman kernels and domains of holomorphy*	
12:15-12:45	Lisa Lorentzen	*Almost all continued fractions converge*	
16:00-16:30	Kristina Krulic Himmelreich	*Generalizations and refinements of Opial type inequalities*	Gunter Semmler
16:35-17:05	Sergei Kalmykov	*Rational Bernstein- and Markov-type inequalities*	
17:10-17:40	Mihaela Ribicic Penava	*Weighted Ostrowski and Grüss type inequalites with applications*	
17:45-18:15	Mario Krnic	*More accurate Jensen-type inequalities obtained via linear interpolation and applications*	

Session 5, Room F VI, Building C

Time	Speaker	Title	Chair
10:30-11:00	Gholam Reza Hojjati	*Multistage-multivalue methods with inherent stability property for ordinary differential equations*	Winfried Auzinger
11:05-11:35	Seyyed Ahmad Hosseini	*On the numerical solution of nonlinear systems of Delay Volterra integro-differential equations with constant delay*	
11:40-12:10	Mohammad Javidi	*A new numerical differentiation formula to approximate the fractional differential equations*	
12:15-12:45	Hossein Kheiri	*Synchronization and anti-synchronization of fractional order chaotic systems and their application in secure communication*	
16:00-16:30	M. Manuela Rodrigues	*Some results concerning the fundamental solution for the time-fractional telegraph equation in higher dimensinos*	Roger Barnard
16:35-17:05	Nelson Vieira	*Fundamental solution of the time-fractional telegraph Dirac operator*	
17:10-17:40	Ali Abdi Kalasour	*Geometric second derivative numerical methods for solving Hamitonian problems*	
17:45-18:15	Hossein Pourbashash	*On the solution of time fractional mobile/immobile equation using spectral collocation method*	

Thursday, July 13

Session 1, Room 105, Building D

Time	Speaker	Title	Chair
10:30-11:00	Marina Makhmutova	*Spherical derivative and closeness of a-points*	Norman Levenberg
11:05-11:35	Arnold Kowalski	*On deviations and spreads of meromorphic minimal surfaces*	
11:40-12:10	Kazuya Tohge	*Meromorphic functions that share four or five pairs of values*	
12:15-12:45	Predrag Vuković	*Refinements of discrete Hilbert-type inequalities*	

Session 2, Room 156, Building C

Time	Speaker	Title	Chair
10:30-11:00	Hirokazu Shimauchi	*Visualizing the solution of the radial Loewner equation*	Adam Lecko
11:05-11:35	Ikkei Hotta	*Tightness results for infinite-slit limits of the chordal Loewner equation*	
11:40-12:10	Samiel Krushkal	*Extended Schwarz lemma, complex geodesics and bounds for coefficients of holomorphic functions*	
12:15-12:45	Iason Efraimidis	*On Bombieri's conjecture for univalent functions*	

Session 3, Room 217, Building D

Time	Speaker	Title	Chair
10:30-11:00	Béla Nagy	*Some results on open-up with overview and applications*	Arno Kuijlaars
11:05-11:35	Hidenori Ogata	*Hyperfunction method for numerical integration and Fredholm integral equations of the second kind*	
11:40-12:10	Antti Rasila	*Conformal modulus and numerical conformal mappings*	
12:15-12:45	Hasi Wulan	*Distance from Bloch type functions to Q_K spaces*	

Session 4, Room 332, Building D

Time	Speaker	Title	Chair
10:30-11:00	Richard Fournier	*On two interpolation formulas*	Stephan Ruscheweyh
11:05-11:35	Nikos Stylianopoulos	*Strong asymptotics for Szegö polynomials for non-smooth curves*	
11:40-12:10	Vladimir Andrievskii	*On polynomial inequalities in the complex plane*	
12:15-12:45	Federico Piazzon	*Pluripotential numerics*	

Poster Session, First Floor Hall, Building D

Arzu Akgül, *On quasi subordination for analytic and biunivalent function class.*

Rabia Aktas, *A study on harmonic functions associated with Jacobi polynomials on the triangle.*

Şahsene Altinkaya, *On the coefficient problems for a new subclass of bi-univalent functions.*

Dimitrios Askitis, *Complete monotonicity of ratios of products of entire functions.*

Yelda Aygar, *Spectral analysis of a selfadjoint difference equation in quantum calculus.*

Asena Çetinkaya, *q- Properties of close-to-convex functions.*

Olena Chumak, *Approximation of periodic functions of hight smoothness by rectangle Fourier sums.*

Joanna Jurasik, *Asymmetric truncated Toeplitz operators on finite-dimensional spaces.*

Seong-A Kim, *Invariant families under a starlike preserving transformation.*

Masashi Kisaka, *Julia sets appear quasiconformally in the Mandelbrot set.*

Marta Kosek, *On an IFS on the space of pluriregular compact subsets of* $\mathbf{C}^N$.

Marijan Markovic, *The Khavinson conjecture.*

Małgorzata Michalska, *Matrix representations of truncated Hankel operators.*

Rozarija Mikic, *Converses of the Edmundson-Lah-Ribarič inequality for Shannon entropy and Csiszár divergence with applications to Zipf-Mandelbrot law.*

Josiah Park, *Asymptotics for Steklov eigenvalues on non-smooth domains.*

Hadi Roopaei, *The generating function of Cesáro sequence and its applications.*

Patanjali Sharma, *Application of Lie transform to Morse oscillator.*

Paweł Sobolewski, *Products of Toeplitz and Hankel operators on the Bergman space in the polydisk.*

Roksolyana Stolyarchuk, *Companion matrices and stability analysis.*

Djordjije Vujadinovic, *Spectral asymptotic of Cauchy's operator on harmonic Berhman space on a simple connected domain and logarithmic potential type operator.*

Paweł Zaprawa, *On a coefficient problem for close-to-convex functions.*

Friday, July 14

Session 1, Room 105, Building D

Time	Speaker	Title	Chair
10:30-11:00	Paul Gauthier	*Approximation by random holomorphic functions*	Franck Wielonsky
11:05-11:35	Sergii Favorov	*On Fourier quasicrystals*	
11:40-12:10	Anders Gustafsson	*Approximation with rational interpolants in $A^{-\infty}(D)$ for Dini domains*	
12:15-12:45	Sina Sadeghi Baghsorkhi	*A new framework for numerical analysis of nonlinear systems: the significance of the Stahl's theory and analytic continuation via Padé approximants*	
16:00-16:30	Renata Rososzczuk	*The failure of Beurling type theorem for A^2_4*	David Kalaj
16:35-17:05	Tomasz Beberok	*L^p-norm estimate of the Bergman projection on the Hartogs triangle*	
17:10-17:40	Maike Thelen	*The backward Taylor shift on Bergman spaces*	
17:45-18:15	Jiří Jahn	*Harmonic Bergman spaces and hypergeometric functions*	

Session 2, Room 156, Building C

Time	Speaker	Title	Chair
10:30-11:00	Juha-Matti Huusko	*Linear differential equations with slowly growing solutions*	Andrzej Sołtysiak
11:05-11:35	Igor Chyzhykov	*Asymptotic behaviour of logarithmic means of analytic functions in the unit disc*	
11:40-12:10	Stephen Deterding	*Bounded point derivations on $R^p(X)$ and approximate derivatives*	
12:15-12:45	Rafał Pierzchała	*Markov's inequality and polynomial mappings*	
16:00-16:30	Ewa Ciechanowicz	*Value distribution and growth of meromorphic solutions of certain Painleve equations*	Shamil Makhmutov
16:35-17:05	Risto Korhonen	*Delay differential Painlevé equations and Nevanlinna theory*	
17:10-17:40	Katsuya Ishizaki	*Meromorphic solutions of some algebraic difference equations in the complex plane*	
17:45-18:15			

Session 3, Room 217, Building D

Time	Speaker	Title	Chair
10:30-11:00	Marvin Müller	*Harmonic Faber polynomials and harmonic Faber expansions*	Michael Dorff
11:05-11:35	María Martín	*A harmonic maps approach to fluid flows*	
11:40-12:10	Stavros Evdoridis	*On geometric properties of polyharmonic mappings*	
12:15-12:45			
16:00-16:30	Pengcheng Wu	*To be announced*	Nikos Stylianopoulos
16:35-17:05	Timothy Opoola	*On subordination associated with certain classes of analytic functions defined by q-derivative operator*	
17:10-17:40	Jugal Kishore Prajapat	*Radius of starlikness and Hardy space of Mittag-Leffler functions*	
17:45-18:15	Pravati Sahoo	*Fekete-Szegö problems for certain class of analytic functions associated with quasi-subordination*	

Session 4, Room 332, Building D

Time	Speaker	Title	Chair
10:30-11:00	Alekos Vidras	*Bergman-Weil expansion for holomorphic functions*	Mark Elin
11:05-11:35	Peter Dragnev	*Biased coin toss and Nash equilibrium*	
11:40-12:10	Myrto Manolaki	*Boundary behaviour of Dirichlet series and applications*	
12:15-12:45	Matthew Fleeman	*Torsional rigidity and Bergman analytic content of simply connected Regions*	
16:00-16:30	Ozge Dalmanoglu	*On the Lupaş q-analogue of the Bernstein operators of Max product kind*	Mikhail Tyaglov
16:35-17:05	Sevilay Kirci Serenbay	*Approximation and shape preserving properties of the Jain operator of Max-product kind*	
17:10-17:40	Samer Alhatemi	*Solving a Laplace equation with Dirichlet-Neumann conditions using integral equation with the adjoint generalized Neumann kernel on multiply connected domains*	
17:45-18:15	Rabia Aktas	*A generalization of two-variable orthogonal functions*	

Plenary lectures

Harmonic Mappings via Real and Complex analysis: Thirty Years after

by

D. Bshouty
Department of Mathematics, Technion, Haifa

The renewed approach to harmonic mappings via real and complex analysis that started with a seminal paper of Clunie and Sheil-Small after the proof of de Branges of the Bieberbach conjecture drew many complex analysts to research the field. I shall present the advancement in the field in four directions: harmonic polynomials, the Riemann mapping theorem for harmonic mappings, univalent and multivalent polygonal mappings, and boundary values.

Exponential polynomials and oscillation theory

by

Janne Heittokangas

University of Eastern Finland and Taiyuan University of Technology

An exponential polynomial of order q is an entire function of the form

$$f(z) = P_1(z)e^{Q_1(z)} + \cdots + P_k(z)e^{Q_k(z)},$$

where the P_j's and Q_j's are polynomials in z such that $\max\{\deg(Q_j)\} = q$. Such a function can be written in the normalized form

$$f(z) = H_0(z) + H_1(z)e^{w_1 z^q} + \cdots + H_m(z)e^{w_m z^q},$$

where the $H_j(z)$'s are either exponential polynomials of order $< q$ or ordinary polynomials in z, the coefficients w_j are pairwise distinct, and $m \leq k$.

The study of exponential polynomials in the case $q = 1$ was initiated independently by Pólya, Ritt and Schwengeler in the 1920's, and since then these functions have been investigated by various authors until today. The general case $q \geq 1$ was considered by Steinmetz in the late 1970's, and by Gackstatter, Meyer and Steinmetz in the early 1980's.

This talk consists of two parts. In the first part we recall some general properties of exponential polynomials, and introduce new results on the value distribution of these functions. In the second part we illustrate how exponential polynomials appear in the complex oscillation theory. Mostly we will concentrate on the second order case

$$f'' + A(z)f = 0,$$

where $A(z)$ is an exponential polynomial, but we will also discuss results on general order linear differential equations. The previous results on complex oscillation theory relevant to this talk were written by Bank, Laine and Langley in the 1980's, by Ishizaki and Tohge in 1997, and by Tu and Yang in 2010.

The conjugate leading coefficients $W = \{\overline{w}_1, \ldots, \overline{w}_m\}$ and the convex hull $\mathrm{co}(W)$ of W play a fundamental role in all discussions.

This presentation is based on a joint work with Gary Gundersen, Katsuya Ishizaki, Ilpo Laine, Kazuya Tohge and Zhi-Tao Wen.

Growth of Painlevé transcendents

by

AIMO HINKKANEN
University of Illinois at Urbana–Champaign

The six Painlevé differential equations were discovered by Painlevé, Fuchs, and Gambier in the period 1895–1910 as those essentially new differential equations in the complex domain of the form $w'' = F(z, w, w')$, where F is meromorphic in z and rational in w and w', whose solutions have no movable singularities other than poles. The solutions, the Painlevé transcendents, may be thought of as non-linear analogues of the classical special functions. The simplest equation is the first one, $w'' = 6w^2 + z$, with no parameters. The other equations involve up to four arbitrary complex parameters.

The Painlevé equations have found applications in numerous situations in mathematics, as well as in physics and other sciences, and engineering. Areas where one of the Painlevé equations, or an equation equivalent to one of them, has made an appearance, include Bonnet surfaces and random matrices in pure mathematics, and general relativity and cosmology, and resonant oscillations in shallow water in other sciences. Properties of special solutions of the equations have been studied in these connections, both theoretically and numerically.

In this talk we discuss properties of the Painlevé transcendents from the point of view of complex analysis.

The Painlevé property states that any local solution of a Painlevé equation can be analytically continued outside the fixed singularities of the equation. In particular, the solutions of the first, second, and fourth equations are single-valued meromorphic functions in the complex plane. We will sketch some aspects of that proof for the first and second equations.

The growth of such single-valued meromorphic solutions can be studied from the point of view of the Nevanlinna theory. Transcendental solutions are of finite order, and in work of several authors over time, the possible orders that may occur have been identified. We survey the growth results obtained for these functions.

This presentation is based on joint work with Ilpo Laine

Universality for conditional measures of the sine point process

by

Arno Kuijlaars
Katholieke Universiteit Leuven, Belgium

The sine process is a random point process that is obtained as a limit from the eigenvalues of many random matrices as the size tends to infinity. This phenomenon is called universality in random matrix theory, and it also holds for many orthogonal polynomial ensembles.

In this talk I want to emphasize another connection of the sine point process with orthogonal polynomials. It comes from a surprising property called number rigidty in the sense of Ghosh and Peres. This means that for almost all configurations, the number of points in an interval $[-R, R]$ is determined exactly by the points outside the interval. The conditional measures is the joint distribution of the points in $[-R, R]$ given the points outside. Bufetov showed that these are orthogonal polynomial ensembles with a weight that comes from the points outside $[-R, R]$.

I will report on recent work with Erwin Mina-Diaz (arXiv:1703.02349) where we prove a universality result for these orthogonal polynomial ensembles that in particular implies that the correlation kernel of the orthogonal polynomial ensemble tends to the sine kernel as R tends to infinity. This answers a question posed by Alexander Bufetov.

Fourier quasicrystals

by

Nir Lev
Bar-Ilan University

By a Fourier quasicrystal we mean a pure point measure whose Fourier transform is also a pure point measure. This notion was inspired by the experimental discovery of quasicrystalline materials in the middle of 80's.

The classical example of such a measure comes from Poisson's summation formula. Which other measures of this type may exist? I will give the relevant background on this problem and present our recent results obtained in joint work with Alexander Olevskii.

Multivariate approximation, convex bodies and pluripotential theory

by

Norm Levenberg
Indiana University

Motivated by a recent paper of T. Bayraktar on random sparse polynomials and on observations of N. Trefethen in multivariate approximation theory, we consider subclasses of the full polynomial space in d dimensions associated to a convex body P in $(\mathbf{R}^+)^d$. In particular, we prove a version of the Bernstein-Walsh theorem on uniform polynomial approximation of holomorphic functions on compact sets in several complex variables in this setting. In addition, we describe how standard notions and results in weighted pluripotential theory generalize (e.g., weighted transfinite diameter) and we obtain asymptotics of weighted Fekete arrays, weighted optimal measures, and weighted Bergman/Christoffel functions.

This presentation is based on a joint work(s) with Turgay Bayraktar, Tom Bloom and Len Bos.

Interior Estimate for elliptic PDE, Quasiconformal and HQC mappings between Lyapunov Jordan domains

by

Miodrag Mateljević
University of Belgrade

We prove that if h is a quasiconformal (shortly qc) mapping of the unit disk $\mathbb{U}$ to a Lyapunov domain, then it maps subdomains of Lyapunov type of $\mathbb{U}$, which touch the boundary of $\mathbb{U}$, onto domains of similar type. We can regard this result as "good local approximation of the qc mapping h by its restriction to a special Lyapunov domain so that its codomain is "locally convex". In particular if h is a hqc (harmonic qc) mapping of $\mathbb{U}$ onto Lyapunov, using it, this shows that h is co-Lip on $\mathbb{U}$. This settles an open intriguing problem in the subject and can be regarded as a version of the Kellogg- Warschawski theorem for hqc.

We also study the growth of the gradient of mappings which satisfy certain PDE equations (or inequalities) using the Green-Laplacian formula for functions and its derivatives. If, in addition, the considered mappings are quasiconformal (qc) between C^2 domains, we show that they are Lipschitz.

This presentation is partially based on joint work with Vladimir Bozin.

Analysis for Dirichlet series and the Riemann zeta function

by

Kristian Seip
Norwegian University of Science and Technology

We have in recent years seen a notable growth of interest in certain functional analytic aspects of the theory of ordinary Dirichlet series

$$\sum_{n=1}^{\infty} a_n n^{-s}.$$

Inspired by the classical theory of Hardy spaces and the operators acting on such spaces, this topic is also intertwined with analytic number theory and function theory on the infinite dimensional torus. Of particular interest are problems that involve an interplay between the additive and multiplicative structure of the integers, in this context embodied respectively by function theory in half-planes and the so-called Bohr lift that transforms Dirichlet series into functions of infinitely many complex variables.

In this survey talk, I will present some highlights from the function theory of Hardy spaces of Dirichlet series, outline some aspects of the operator theory that has been developed for these spaces, and present some applications to the Riemann zeta function $\zeta(s)$, including pseudomoments and lower bounds for the growth of $\zeta(1/2 + it)$.

Szegö-Widom Asymptotics for Chebyshev Polynomials on Subsets of $\mathbb{R}$

by

Barry Simon
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Chebyshev polynomials for a compact subset $\mathfrak{e} \subset \mathbb{R}$ are defined to be the monic polynomials with minimal $\| \cdot \|_\infty$ over $\mathfrak{e}$. In 1969, Widom made a conjecture about the asymptotics of these polynomials when $\mathfrak{e}$ was a finite gap set. We prove this conjecture and extend it also to those infinite gap sets which obey a Parreau-Widom and a Direct Cauchy Theory condition. This talk will begin with a generalities about Chebyshev Polynomials.

This presentation is based on a joint work with Jacob Christiansen and Maxim Zinchenko and partly with Peter Yuditskii.

Critical points of polynomials

by

Vilmos Totik
University of Szeged and University of South Florida

The Gauss-Lucas theorem says that if P is a polynomial, then the zeros of the derivative P' (i.e. the critical points of P) lie in the convex hull of the zeros of P. This talk will be about the location of the critical points within this convex hull. The main emphasis will be to determine the asymptotic distribution of the critical points, and when that distribution is the same as the distribution of the zeros. There are examples (like $P_n(z) = z^n - 1$) when these two distributions are not the same, but it will be shown that they are extremely unstable.

Invited lectures

Geometric second derivative numerical methods for solving Hamiltonian problems

by

A. Abdi
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In this paper we construct second derivative methods for the efficient solution of Hamiltonian systems. Since some special properties are required for the good long-time behavior of a numerical method applied to Hamiltonian problems, we are seeking some sufficient conditions on the second derivative general linear methods (see, e.g., [1, 2, 3, 4]) to preserve the geometric behavior of the solution [5]. Implementation of the constructed methods on the well-known Hamiltonian problems such as outer solar problem confirms their efficiency.

Keywords: Hamiltonian systems, Geometric integration, General linear methods, Second derivative methods.

This presentation is based on a joint work with G. Hojjati and S. A. Hosseini

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A generalization of two-variable orthogonal functions

by

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In the paper [2], Koornwinder constructed a method to derive orthogonal polynomials in two variables from orthogonal polynomials in one variable. As an application of this method, he defined various two-variable orthogonal polynomials. One of these are orthogonal polynomials on the triangle. Inspired by the orthogonal polynomials on the triangle, in this paper we introduce a family of two-variable orthogonal functions. We obtain several recurrence relations and we derive generating functions for these functions. Furthermore, we present some special cases of these functions.

This presentation is based on a joint work with Guner Ozturk and Fatma Tasdelen

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Solving a Laplace equation with Dirichlet-Neumann conditions using integral equation with the adjoint generalized Neumann kernel on multiply connected domains

by

S. A. A. Al-Hatemi
Imam Abdulrahman Bin Faisal University IAU, Saudi Arabia

In this paper, a new boundary integral equation method is developed for solving Laplace's equation with mixed boundary conditions in both bounded and unbounded multiply connected domains. The method is based on a uniquely solvable boundary linear integral equation with the adjoint generalized Neumann kernel. The mixed boundary value problems are reformulated as an adjoint Riemann–Hilbert boundary value problem. The adjoint Riemann–Hilbert problem is then solved using a uniquely solvable Fredholm integral equations with the adjoint generalized Neumann kernel. Several numerical examples are presented to illustrate the efficiency of the method yielding a high accuracy.

On Uniform Bounds for Complex Polynomials

by

V. Andrievskii
Kent State University, USA

We discuss the following two topics concerning algebraic polynomials.

First, the estimates of the uniform norm of the Chebyshev polynomials associated with a compact set K in the complex plane are considered. These estimates are exact (up to a constant factor) in the case where K consists of a finite number of quasiconformal curves or arcs. The case where K is a uniformly perfect subset of the real line is also studied.

Second, we present the exact (up to the constants) double inequality for the Christoffel function for the generalized Jacobi measure supported on a Jordan domain bounded by a quasiconformal curve. Note that the quasiconformality of the boundary cannot be omitted.

Adaptive integration of nonlinear evolution equations

by

Winfried Auzinger
Technische Universität Wien, Austria

We give an overview on recent work on a posteriori error estimation and time-adaptive integration of nonlinear evolution equations of parabolic and Schrödinger types. In particular, splitting techniques combined with spectral approximations in space are considered. The computation of a posteriori local error estimates can be based on optimized embedded pairs of schemes, the use of the adjoint of an (optimized) scheme, or defect-based evaluations. We discuss the construction and implementation of such error estimators and present some examples.

This presentation is based on a joint work with Harald Hofstätter, Othmar Koch, and Michael Quell.

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On the spectral analysis of a discrete Sturm—Liouville equation

by

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During the last decades, authors have paid attention to investigate the spectral analysis of different kinds of equations and operators. Some of them are the Sturm-–Liouville equation, Schrödinger equation and Dirac equation. In general, Jost solution is important to get the spectral properties of these equations. There are a lot of studies about the spectral analysis of discrete Sturm—Liouville equations both selfadjoint and non-selfadjoint cases which have exponential-type Jost solution in general. Only a few works give information about the spectral analysis by using the properties of polynomial-type Jost solution, but none of them consist spectral parameter in boundary condition. Differently from these studies, in this paper, we consider a boundary value problem (BVP) consisting of a discrete Sturm-Liouville equation and boundary conditions depending on a spectral parameter. We find the polynomial-type Jost solution of this BVP and using the analytical properties and asymptotic behavior of this Jost solution, we investigate the spectrum of this BVP. Discussing the point spectrum, we present a condition that guarantees that this BVP has a finite number of eigenvalues and spectral singularities with finite multiplicities.

This presentation is based on a joint work with Elgiz Bairamov and G. Gülçehre Özbey.

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Growth estimate of entire functions of bounded L-index in joint variables

by

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We use definitions and notations from [1]. Our main result is

Theorem 1. *Let $\mathbf{L}(Re^{i\Theta})$ be a positive continuously differentiable function in each variable $r_k \in [0,+\infty)$, $k \in \{1,\dots,n\}$, $\Theta \in [0,2\pi]^n$. If an entire function F has bounded $\mathbf{L}$-index $N = N(F,\mathbf{L})$ in joint variables then for every $\Theta \in [0,2\pi]^n$ and for every $R \in \mathbb{R}^n_+$ ($r_m \neq 0$) and $K \in \mathbb{Z}^n_+$*

$$\begin{aligned} &\ln \max\left\{ \frac{|F^{(K)}(Re^{i\Theta})|}{K!\mathbf{L}^K(Re^{i\Theta})} : \ \|K\| \le N \right\} \\ &\quad \le \ln\max\left\{ \frac{|F^{(K)}(\mathbf{0})|}{K!\mathbf{L}^K(\mathbf{0})} : \ \|K\| \le N \right\} \\ &\qquad + \int_0^{r_m} \left(\max_{\|K\|\le N} \left\{ \sum_{j=1}^n \frac{r_j}{r_m}(k_j+1) l_j\left(\frac{\tau}{r_m} Re^{i\Theta}\right) \right\} \right. \\ &\qquad\qquad \left. + \max_{\|K\|\le N} \left\{ \sum_{j=1}^n \frac{k_j(-u_j'(\tau))^+}{l_j\left(\frac{\tau}{r_m}Re^{i\Theta}\right)} \right\} \right) d\tau, \end{aligned} \tag{1}$$

If, in addition, there exists $C > 0$ such that the function $\mathbf{L}$ satisfies inequalities

$$\sup_{R\in\mathbb{R}^n_+} \max_{t\in[0,r_m]} \max_{\Theta\in[0,2\pi]^n} \max_{1\le j\le n} \frac{(-(u_j(t,R,\Theta))'_t)^+}{\frac{r_j}{r_m} l_j^2(\frac{t}{r_m}Re^{i\Theta})} \le C, \tag{2}$$

$$\max_{\Theta\in[0,2\pi]^n} \int_0^{r_m} \sum_{j=1}^n \frac{r_j}{r_m} l_j\left(\frac{\tau}{r_m}Re^{i\Theta}\right) d\tau \to +\infty \ \text{as} \ \|R\| \to +\infty \tag{3}$$

then

$$\overline{\lim_{\|R\|\to+\infty}} \frac{\ln \max\{|F(z)\colon\ z \in T^n(\mathbf{0},R)\}}{\max\limits_{\Theta\in[0,2\pi]^n} \int_0^{r_m} \sum_{j=1}^n \frac{r_j}{r_m} l_j\left(\frac{\tau}{r_m}Re^{i\Theta}\right) d\tau} \leq (C+1)N+1. \tag{4}$$

And if $r_m(-(u_j(t,R,\Theta))'_t)^+/(r_j l_j^2(\frac{t}{r_m}Re^{i\Theta})) \to 0$ *uniformly for all* $\Theta \in [0,2\pi]^n$, $j \in \{1,\dots,n\}$, $t \in [0,r_m]$ *and* (3) *holds as* $\|R\| \to +\infty$ *then*

$$\overline{\lim_{\|R\|\to+\infty}} \frac{\ln \max\{|F(z)\colon\ z \in T^n(\mathbf{0},R)\}}{\max\limits_{\Theta\in[0,2\pi]^n} \int_0^{r_m} \sum_{j=1}^n \frac{r_j}{r_m} l_j\left(\frac{\tau}{r_m}Re^{i\Theta}\right) d\tau} \leq N+1. \tag{5}$$

This presentation is based on a joint work with Prof. O. B. Skaskiv.

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Applications of a "Two Point Boundary Variation"

by

Roger Barnard
Texas Tech University , United States

In 2004 Alex Solynin and I developed a "Two Point Boundary Variation Formula" for the change in the reduced module of a domain in the complex plane with minimal assumptions on the boundary. In this talk we will discuss this variation and how we have been able to combine it with polarization, symmetrization, and quadratic differential methods to solve a variety of extremal problems over the last dozen years. These include determining sharpened isometric inequalities for the logarithmic capacity of both convex and non-convex domains involving their diameter and area, determining an isometric inequality for hyperbolic polygons using our lemma verifying the convexity of a complicated combination of gamma functions, solving the minimum area problem for non-vanishing functions in the plane, and solving iceberg-type problems involving estimating hidden parts of a variety of domains from their visible parts.

Real entire functions, zero-location and structured minors with applications to the Riemann hypothesis and Turan's inequalities

by

Prashant Batra

Hamburg University of Technology; Hamburg, Germany.

For real entire functions with exclusively negative zeros important connections to totally non-negative (TNN) matrices exist. Characterization of those entire functions whose Taylor expansion $\sum a_k x^k, a_0 > 0$, generates totally non-negative matrices $(a_{j-i})_{i,j=0}^{\infty}$, is a consequence of the AESW-theorem (theorem of Aissen, Edrei, Schoenberg, and Whitney). The structured matrices $H(g,h)$, exhibited by Hurwitz in his redraft of the Hermite-Jacobi approach to quadratic forms, are known to have all minors non-negative if and only if the polynomial $f(z) = h(z^2) + zg(z^2)$ with positive coefficients has all zeros in the closed left half-plane, i.e., if and only if (g,h) is a generalized positive pair with positive coefficients. Dyachenko (2014) gave a complete characterization of those series generating a certain TNN super-matrix $\hat{H}$ of H. In connection with transforms of the Riemann ξ-function the TNN minors yield a set of coefficient inequalities (exponentially growing with the number of considered coefficients) bearing on the Riemann hypothesis.

We obtain a transcendental, computational characterization of correctness of the Riemann hypothesis involving only *essential* minors (improving over related approaches of Nuttall and Dyachenko). We show that the Laguerre-Turán inequalities (involving three consecutive coefficients) discussed in this connection since Pólya's question of 1927, and the successive four term improvement by Craven-Csordas (2002), are a weaker necessary criterion for the Riemann hypothesis than the four term inequality from the first non-trivial of our essential minors.

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Universality principles for zeros of random polynomials

by

Turgay Bayraktar
University of Hartford

I will present several universality principles concerned with zero distribution of random polynomials. In one direction, universality phenomenon indicates that under natural assumptions, asymptotic distribution of appropriately normalized zeros of random polynomials is independent of the choice of probability law defined on random polynomials. Another form of universality is asymptotic normality of smooth linear statistics of zero currents. Finally, if time permits, I will also describe some recent results on universality of scaling limits of correlations between simultaneous zeros of random polynomials.

L^p-norm estimate of the Bergman projection on the Hartogs triangle

by

Tomasz Beberok

University of Agriculture in Krakow

The purpose of this talk is to give an estimate of the L^p-norm of the Bergman projection on the Hartogs triangle, the pseudoconvex domain in $\mathbb{C}^2$ defined as

$$\mathcal{H} = \left\{ (z_1, z_2) \in \mathbb{C}^2 \colon |z_1| < |z_2| < 1 \right\},$$

for $4/3 < p < 4$. The Bergman space $L^2_h(\mathcal{H})$ is defined to be the intersection $L^2(\mathcal{H}) \cap \mathcal{O}(\mathcal{H})$ of the space $L^2(\mathcal{H})$ of square integrable functions on $\mathcal{H}$ with the space $\mathcal{O}(\mathcal{H})$ of holomorphic functions on $\mathcal{H}$. The orthogonal projection operator $\mathbf{P} \colon L^2(\mathcal{H}) \to L^2_h(\mathcal{H})$ is the Bergman projection associated with the domain $\mathcal{H}$. In [1], Chakrabarti and Zeytuncu proved that the Bergman projection $\mathbf{P}$ is a bounded operator from $L^p(\mathcal{H})$ to $L^p_h(\mathcal{H})$ if and only if $4/3 < p < 4$. A natural and interesting question is to determine the exact value of the L^p-operator norm $\|\mathbf{P}\|_p$ of this operator. This turns out to be a difficult task to accomplish, except for the trivial case when $p = 2$. Our main result reads as follows.

Theorem. *For $4/3 < p < 4$, we have*

$$\Gamma^2\left(\frac{2}{p}\right)\Gamma^2\left(\frac{2}{q}\right) \le \|\mathbf{P}\|_p \le (M(p))^{\frac{1}{q}}\,(M(q))^{\frac{1}{p}},$$

where $M(t) = \Gamma\left(1 - \frac{4}{3t}\right)\Gamma\left(\frac{4}{3t}\right)$ and $q := \frac{p}{p-1}$ is the conjugate exponent of p.

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On Some New Results Related to Special Functions

by

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In this work we present new results involving generalized Struve function.

This presentation is based on a joint work with Daniel Breaz

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New asymptotic expansions and improvements of approximation formulas for the gamma function

by

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The well-known Stirling approximation for the factorial function

$$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

is one of the most beautiful formulas in mathematics. This is in fact a shortening of the asymptotic expansion for the gamma function which was also studied in other similar forms by Laplace, De Moivre, Ramanujan, Wehmeier and recently by Karatsuba, Gosper, Batir, Mortici, Nemes and others.

But all these formulas have been considered separately and connection between them wasn't clear. Moreover, the computation of each term in thiese formulas was a tedious job without any attempt to find a general procedure to calculate coefficients in this type of asymptotic expansions.

In this talk we present a general expansion for the gamma function introducing a parameter m. Using properties of asymptotic power series, we proved an asymptotic expansion for the factorial function

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \left[\sum_{k=0}^{\infty} P_k\, n^{-k}\right]^{1/m},$$

where coefficients (P_n) satisfy a simple recursive algorithm.

This allows an easy calculation of the coefficients in all of the previously mentioned expansions and leads to various other generalizations and improvements of the approximation formulas for the gamma function and related classical functions.

Some generalizations of the fundamental normality test

by

K. S. Charak

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The theory of normal families of meromorphic functions initiated by Paul Montel in 1907 now forms an integral part of function theory. In fact, this theory is responsible for many exciting advances in the area of complex dynamics, but there has also been many far reaching internal developments in the theory during the last over hundred years. My lecture is an attempt to illustrate how the developments have taken place and still continue to develop by concentrating on one particular result-the Fundamental Normality Test: *if each member of a family of meromorphic functions on some domain omit three distinct values of the extended complex plane, then the family is normal.* In fact, I will try to explore some of the directions in which this result has been extended and generalized over a little less than hundred years. Also, I shall present some of my contributions(jointly with Virender Singh) in this direction.

When does a formal finite-difference expansion become ”real”?

by

Edmund Y. M. Chiang
Hong Kong University of Science & Technology

Let $\Delta f(x) = f(x+1) - f(x)$. A basic finite difference operational calculus formula states that $\Delta^n f(x) = (e^D - 1) f(x)$ where $D = \frac{d}{dx}$ is understood in a formal manner. We show that such formula becomes possible when applied to slow growth meromorphic functions in $\mathbf{C}$. We then apply this result to give estimates to entire solutions of slower growth of linear difference equations with polynomial coefficients. This presentation is based on a joint work with Shao-Ji Feng.

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Quadratic differentials related to Heun equations.

by

Mondher Chouikhi
ISSAT Gabes.Tunisia, Tunisia

In this talk, we discuss the existence of solutions (as a Cauchy transform of signed measures) of two algebraic equations : the first is $\mathcal{C}^2 + r(z)\mathcal{C} + s(z) = 0$, where r and s are polynomials of degrees 2 and 1 respectively; the second is $t(z)\mathcal{C}^2 + r(z)\mathcal{C} + s(z) = 0$, where t is a polynomial of degree 1 and r, s are as in the first equation. These algebraic equations are obtained from a Shroedinger equation with a ”quartic” and ”sextic” PT-symmetric potential after an appropriate re-scaling of the spectrum of Hamiltonian associated to the Shroedinger equation. These problems remain to describe the critical graph of a related polynomial and rational quadratic differentials which will be described.

Asymptotic behaviour of logarithmic means of analytic functions in the unit disc

by

Igor Chyzhykov
Ivan Franko National University of Lviv, Ukraine

Let SH^∞ be the class of subharmonic functions bounded from above in the unit disc $\mathbb{D}$. If $u \in SH^\infty$ such that $u(z) \le 0$ and $u(z)$ is harmonic in a neighborhood of the origin, then

$$u(z) = \int_{\mathbb{D}} \ln \frac{|z-\zeta|}{|1-z\bar{\zeta}|} d\mu_u(\zeta) - \frac{1}{2\pi} \int_{\partial\mathbb{D}} \frac{1-|z|^2}{|\zeta - z|^2} d\psi(\zeta),$$

where μ_u is the Riesz measure of u, ψ is a Borel measure on $\partial\mathbb{D}$.

For a Borel subset $M \subset \overline{\mathbb{D}}$ such that $M \cap \partial\mathbb{D}$ is Borel measurable on $\partial\mathbb{D}$ the *complete measure* λ_u *of* u *in the sense of Grishin* λ_u is defined by

$$\lambda_u(M) = \int_{\mathbb{D}\cap M} (1-|\zeta|)\, d\mu_u(\zeta) + \psi(M \cap \partial\mathbb{D}).$$

For $p \ge 1$ we define

$$m_p(r,u) = \left(\frac{1}{2\pi} \int_0^{2\pi} |u(re^{i\theta})|^p \, d\theta \right)^{\frac{1}{p}}, \quad 0 < r < 1.$$

Criteria for boundedness of pth integral means, $1 \le p < \infty$, of $\log|B|$ and $\log B$ are established by Ya.V. Mykytyuk and Ya.V. Vasyl'kiv in 2000.

Let $C(\varphi, \delta) = \{\zeta \in \overline{\mathbb{D}} : |\zeta| \ge 1-\delta, |\arg\zeta - \varphi| \le \pi\delta\}$.

Theorem. *Let $f \in H^\infty$, $\gamma \in (0,2)$, $p \in [1,\infty)$. Let λ be the complete measure of* $\log|f|$. *In order that*

$$m_p(r, \log|f|) = O((1-r)^{\gamma-1}), \quad r \uparrow 1,$$

hold it is necessary and sufficient that

$$\left(\int_0^{2\pi} \lambda^p(C(\varphi,\delta))\, d\varphi \right)^{\frac{1}{p}} = O(\delta^\gamma), \quad 0 < \delta < 1.$$

We also prove sharp upper estimates of pth means of analytic and subharmonic functions of finite order in the unit disc. A multidimensional counterpart for M-subharmonic functions in the unit ball in $\mathbb{C}$ is proved as well.

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Value distribution and growth of meromorphic solutions of certain Painlevé equations

by

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Painlevé equation P_1 is connected by the Hamiltonian structure with a second order ordinary differential equation, called a σ-equation S_1, of the form

$$(\sigma'')^2 - 4(\sigma')^3 + 2z\sigma' - 2\sigma = 0.$$

Painlevé equation P_2, on the other hand, is connected by the Hamiltonian structure with the equation P_{34} :

$$f'' = \frac{(f')^2}{2f} + Bf(2f - z) - \frac{A}{2f}, \qquad (A, B \in \mathbb{C})$$

and with a σ-equation S_2 :

$$(\sigma'')^2 + 4(\sigma')^3 + 2\sigma'(z\sigma' - \sigma) = \frac{1}{4}(\alpha + \frac{1}{2})^2, \qquad (\alpha \in \mathbb{C}).$$

Thus the solutions of S_1, P_{34} and S_2 are meromorphic in $\mathbb{C}$. The talk shall cover various value distribution and growth properties of the equations, including order, estimates of defects, multiplicity indices and Petrenko's deviations of transcendental meromorphic solutions. Moreover, behavior towards small target functions shall be discussed.

This presentation is partially based on a joint work with Galina Filipuk

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On the Lupaş q-Analogue of the Bernstein Operators of Max Product Kind

by

Özge Dalmanoğlu

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The Korovkin type approximation theory is based on the study of linear and positive operators. The sum and the product of reals are the operations that are used in this approximation process. Recently an open problem is arose in which it is asked whether all operators have to be linear and summation and multiplication are the only operations that can be used in the approximation theory. As an answer to this question Bede et. al. (see [1]) presented “max-product kind operators” which are non-linear and constructed by using ”maximum” operation instead of the ”sum” operation. They studied the approximation properties of these new operators and gave Jackson-type error estimates in terms of the modulus of continuity. In this paper we dealed with a nonlinear operator of max-product kind based on the q-integers. We constructed Lupaş q-analogue of the Bernstein operators of max product kind and estimate the order of approximation for these operators.

This presentation is based on a joint work with Sevilay Kırcı Serenbay

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Interpolation by bounded analytic functions and related questions

by

Arthur Danielyan
University of South Florida, United States

Fatou's interpolation theorem of 1906 is an elementary but important result which states that for any closed set E of measure zero on $|z| = 1$ there exists an element in the disc algebra which vanishes precisely on E. It implies, for instance, the F. and M. Riesz theorem on analytic measures. In this talk we show that Fatou's theorem directly implies: (i) the Rudin-Carleson interpolation theorem; (ii) a theorem of Lohwater and Piranian on the radial limits of bounded analytic functions; and (iii) a known approximation lemma of R. Doss. We also present some new interpolation results in the unit disc, when the interpolation of a continuous function on a closed (or G_δ) set on $|z| = 1$ is required to be done by the limiting values of a bounded analytic function. In particular, we show that for any G_δ set F of measure zero on $|z| = 1$ there exists a function $g \in H^\infty$ non-vanishing in $|z| < 1$ such that: 1) g has non-zero radial limits everywhere on $\{|z| = 1\} \setminus F$; and 2) g has vanishing unrestricted limits at each point of F. This result obviously extends Fatou's theorem from the closed sets to G_δ sets. It implies an affirmative answer to a question proposed by L. Rubel in 1973.

Bounded point derivations on $R^p(X)$ and approximate derivatives

by

Stephen Deterding
University of Kentucky

Let X be a compact subset of the complex plane and let $R_0(X)$ denote the set of all rational functions whose poles lie off X. The function space $R(X)$ is the uniform closure of $R_0(X)$. The space $R^p(X)$ is an important function space that is closely related to $R(X)$. For $1 \leq p < \infty$, $R^p(X)$ is the closure of $R_0(X)$ in the L^p norm. A bounded point derivation on $R(X)$ at a point x_0 is a bounded linear functional D on $R(X)$ such that $D(fg) = D(f)g(x_0) + D(g)f(x_0)$ for all functions f, g belonging to $R(X)$. It is known if there is a bounded point derivation on $R(X)$ at x_0, then every function in $R(X)$ has an approximate derivative at x_0. An approximate derivative is defined in the same way as the usual derivative except that instead of taking the limit in the difference quotient over all of X, the limit is taken over a subset of X with full area density at x_0. One can also define bounded point derivations for $R^p(X)$, although a slightly different definition must be made. In this talk we will introduce bounded point derivations on $R^p(X)$ and discuss when the existence of a bounded point derivation on $R^p(X)$ implies that every function in $R^p(X)$ has an approximate derivative at x_0.

This presentation is based on a joint work with James Brennan

Convolutions of univalent harmonic strip mappings

by

Michael Dorff
Brigham Young University, United States

Let $f_k = h_k + \overline{g}_k$ be the family of harmonic univalent functions with $h_k(z) + g_k(z) = \frac{z}{1-z}$ for $k = 1, 2$. Earlier it was shown that if $f_1 * f_2$ is locally univalent and sense-preserving, then $f_1 * f_2$ is univalent and convex in the direction of the real axis. This resulted in several papers determining conditions under which $f_1 * f_2$ is locally univalent and sense-preserving. In this paper we consider the family of harmonic univalent functions $f_k = h_k + \overline{g}_k$ (where $k = 1, 2$) that are shears of the analytic map $h_k(z) - g_k(z) = \frac{1}{2} \ln \left(\frac{1+z}{1-z} \right)$ with dilatation $\omega_k = e^{i\theta_k} z^k$. We prove that if the convolution $f_1 * f_2$ is locally one-to-one and sense-preserving, then $f_1 * f_2$ is univalent and convex in the direction of the real axis.

Biased coin toss and Nash equilibrium

by

Peter Dragnev
Indiana University-Purdue University Fort Wayne

This is an approximation theory/game theory approach to a biased coin game ($P(heads) = p$) with two players, where the coin is tossed n times. One of the players chooses $n + 1$ numbers a_i to estimate the p based on each outcome observed. The penalty payoff is $|p - a_i|$. The player tries to minimize the expected payoff. The second player tries to select p that will maximize this expected payoff regardless of the choice a_i of the first. Best strategies satisfy Nash equilibrium and the maximum likelihood is not the top strategy.

This presentation is based on a joint work with David Benko, Dan Coroian and Ramon Orive

One helpful property of functions generating Pólya frequency sequences

by

Alexander Dyachenko
TU-Berlin

It is well-known that the zeros of the Jacobi Theta-function $\sum_{n=-\infty}^{\infty} q^{\frac{n(n-1)}{2}} z^n$ are simple and form the sequence $(-q^j)_{j=-\infty}^{\infty}$ as soon as $0 < |q| < 1$. Something similar can be numerically observed for certain functions of the form

$$F(z;q) = \sum_{n=0}^{\infty} a_n q^{\frac{n(n-1)}{2}} z^n, \quad \text{where} \quad 0 < |q| < 1 \quad \text{and} \quad a_n \geq 0,$$

including the Partial Theta function with $a_n = 1$ (provided that $|q|$ is below certain known value $q_* < 1$) and the "Disturbed Exponential" with $a_n = \frac{1}{n!}$; both appear in problems of statistics and combinatorics. If q is small or real, the zeros of $F(z;q)$ are simple and distinct in absolute value under reasonable conditions on a_n. During my talk, these properties of zeros will be shown for purely imaginary q.

More specifically, we will obtain a more general fact. Let univariate entire functions $f(z)$ and $g(-z)$ have genus 0 and only negative zeros. Then the zeros of $f(z^k)+z^p g(z^k)$ and $g(z^k)+z^p f(z^k)$ are simple (or at most double in a special case) and distributed "uniformly" among $2k$ sectors of the complex plane. This fact extends to the case when $f(z)$ and $g(-z)$ generate Pólya frequency sequences (one- or two-way infinite) with almost no changes. The proof rests on a connection to the Nevanlinna class of functions mapping the upper half of the complex plane into itself.

Multiplier algebras of weighted shifts on directed trees

by

Piotr Dymek
University of Agriculture in Krakow

We study a class of bounded weighted shifts on directed trees with roots focusing on analytic aspects of their theory. We define and study multiplier algebras related to these operators. We show that these algebras consists of coefficients of analytic functions and forms a Banach algebra closed in SOT and WOT topology. From this fact we deduce reflexivity of those weighted shifts on directed trees whose all path-induced spectral-like radii are positive.

This presentation is based on a joint work with P. Budzyński, A. Płaneta and M. Ptak

On Bombieri's conjecture for univalent functions

by

Iason Efraimidis
Universidad Autónoma de Madrid

Let S denote the class of normalized univalent functions in the unit disk. In 1967 Bombieri conjectured in very precise terms what the behavior of the coefficients of functions in S should be close to the Koebe function $K(z) = \frac{z}{(1-z)^2}$. Namely, he proposed that the two real numbers

$$\sigma_{mn} := \liminf_{f \to K} \frac{n - \operatorname{Re} a_n}{m - \operatorname{Re} a_m} \qquad \text{and} \qquad B_{mn} := \min_{t \in \mathbb{R}} \frac{n \sin t - \sin(nt)}{m \sin t - \sin(mt)}$$

should be equal for all $m, n \geq 2$. Although it is known that $0 \leq \sigma_{mn} \leq B_{mn}$ and that $\sigma_{mn} = B_{mn}$ when K is approached only by functions with real coefficients, the Bombieri conjecture has been disproved by Greiner and Roth (2001) in the case $(m,n) = (3,2)$, while disproofs for the points $(2,4), (3,4)$ and $(4,2)$ were then furnished by Prokhorov and Vasil'ev (2005).

Recently, Leung used a second variation formula for the Koebe function to prove that the conjecture is false at the points $(m,2)$ for every $m \geq 3$, and, also, at $(m,3)$ for every odd $m \geq 5$. Complementing his work we shall prove that the conjecture is false in many more points (m,n) that lie in some sectors. The proof will be mainly based on trigonometry.

Fixed points of holomorphic mappings

by

Mark Elin
ORT Braude College

Let $\mathbb{B}$ be an open unit ball in a complex Banach space X. In this talk we discuss conditions that ensure that the set $\mathbb{B} \cap F(\mathbb{B})$ contains the fixed point set of F. A simplest sufficient condition for $F \in \mathrm{Hol}(\mathbb{B}, X)$ to have an interior fixed point in $\mathbb{B}$ consists of the invariance condition

$$F(\mathcal{D}) \subset \mathcal{D} \quad \text{and} \quad \sup_{x \in \mathcal{D}} \left\| F'(x) \right\| < 1 \tag{1}$$

for some closed convex subset $\mathcal{D} \subset \mathbb{B}$. However, condition (1) is too strong and constrains us to use them for solving other problems.

A breakthrough in this direction was done by Earle and Hamilton [1]. However (even in the one-dimensional case) the assumptions of the Earle-Hamilton Theorem become not applicable if F does not map the open unit ball $\mathbb{B}$ into itself. Moreover, many examples show that F might be unbounded on $\mathbb{B}$ but still have a unique fixed point inside.

The goal of our work is to assign such conditions that provide the existence and uniqueness of the fixed point in the open unit ball $\mathbb{B}$ of a general Banach space X for a mapping $F \in \mathrm{Hol}(\mathbb{B}, X)$, which even is not necessarily bounded on $\mathbb{B}$.

This presentation is based on a joint work with David Shoikhet

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On Geometric Properties of Polyharmonic Mappings

by

S. Evdoridis
Aalto University, Finland

In this talk, we establish a univalence criterion for planar polyharmonic mappings that generalises the earlier result of Bharanehdar and Ponnusamy. We also show that there does not exist a univalent, polyharmonic mapping of the unit disc onto the whole complex plane. This result is a generalisation of the classical T. Radó's Theorem.

This presentation is based on a joint work with A. Rasila

On Fourier quasicrystals

by

S. Favorov
Karazin's Kharkiv National University

Let a measure $\mu = \sum_{\lambda\in\Lambda} a_\lambda \delta_\lambda$ be a tempered distribution on $\mathbf{R}^d$ and their Fourier transform $\hat{\mu} = \sum_{\gamma\in\Gamma} b_\gamma \delta_\gamma$ be slowly increasing measure on $\mathbf{R}^d$ with countable Λ, Γ such that $\inf_{\lambda\in\Lambda} |a_\lambda| > 0$. We prove that the discreteness of the set of differences $\Lambda - \Lambda$ implies that the Λ is a finite union of translates of a single full-rank lattice L. Note that here we need not the discreteness of spectra Γ of the measure.

Also, we get a corresponding result for pair of measures μ_1, μ_2 and the difference $\Lambda_1 - \Lambda_2$ of their supports.

Next, let $\mu = \sum_{\lambda\in\Lambda} a_\lambda \delta_\lambda$ be a measure with uniformly discrete support Λ such that $\inf_{\lambda\in\Lambda} |a_\lambda| > 0$, and its the Fourier transform $\hat{\mu}$ be a measure with countable support and variation $|\hat{\mu}|(B(0,r) = O(r^d)$ as $r \to \infty$. Then Λ is a finite union of translates of several disjoint full-rank lattices (maybe noncommensurable).

The arguments are based on a local analog of Wiener's Theorem on absolutely convergent trigonometric series and theory of almost periodic functions.

Torsional Rigidity and Bergman Analytic Content of Simply Connected Regions

by

Matthew Fleeman
Baylor University

In this talk, we exploit the equality of Bergman analytic content and torsional rigidity of a bounded simply connected domain to develop a new method for calculating these quantities. This method is particularly useful for the case when the region is a polygon. We will present several concrete examples to demonstrate the utility of our ideas.

This presentation is based on a joint work with Brian Simanek

On two interpolation formulas

by

Richard Fournier
Université de Montréal, Canada

I will discuss, from various points of view, two recent interpolation formulas for algebraic polynomials leading to various Bernstein-Markov type inequalities.

This presentation is based on a joint work with Stephan Ruscheweyh.

Approximation by random holomorphic functions

by

Paul M. Gauthier
Université de Montréal

Brown and Schreiber introduced a point of view which could lead to many questions on “random complex analysis.” Andrus and Brown obtained stochastic versions of classical theorems in approximation in a single variable. We propose a similar investigation in several complex variables.

Coefficient bounds for two new subclasses of bi-univalent functions

by

Vasudha Girgaonkar
Walchand College Of Engineering,Sangli, India.

Let A denote the class of functions $f(z)$ given by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

which are normalized and analytic in the open unit disc $U = \{z \in \mathbb{C} : |z| < 1\}$.

In [3] Brannan and Taha introduced the bi-uinvalent functions similar to the subclasses of the $S^*(\alpha)$ and $K(\alpha)$ of starlike and convex functions of order α $(0 < \alpha \leq 1)$ respectively. Further in [1] Babalola defined the class $\mathfrak{L}_\lambda(\beta)$ of λ-pseudo starlike functions of order β in U. Taking into consideration these concepts, we have defined two new subclasses $M_\Sigma(\alpha, \lambda, \beta)$ and $B_\Sigma(\gamma, \lambda, \beta)$ of bi-univalent functions in U and established the estimates on the coefficients $| a_2 |$ and $| a_3 |$.

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Extensions of harmonic functions of the complex plane slit along an interval

by

Armen Grigoryan
The John Paul II Catholic University of Lublin, Poland

Let I be a line segment in the complex plane $\mathbb{C}$. We describe a method of constructing a bilipschitz sense-preserving mapping of $\mathbb{C}$ onto itself which is harmonic in $\mathbb{C} \setminus I$ and coincides with a given sufficiently regular function $f : I \to \mathbb{C}$. As a result we show that a quasiconformal self-mapping of $\mathbb{C}$ which is harmonic in $\mathbb{C} \setminus I$ does not have to be harmonic in $\mathbb{C}$.

This presentation is based on a joint work with Andrzej Michalski and Dariusz Partyka.

Approximation with rational interpolants in $A^{-\infty}(D)$ for Dini domains

by

Anders Gustafsson
Nanyang Technological University, Singapore

Let D denote a Dini domain in $\mathbb{C}$ and let $\overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$. For each $n = 1, 2, 3 \ldots$, take points $A_n = \{a_{ni}\}_{i=0}^{n}$ in D and points $B_n = \{b_{ni}\}_{i=1}^{n}$ in $\overline{\mathbb{C}} \setminus D$. Let α_n and β_n be the normalized point counting measures of A_n and B_n. Suppose that $\alpha_n \xrightarrow{w^*} \alpha$, $\beta_n \xrightarrow{w^*} \beta$ and denote by α' and β' their swept measures onto ∂D. Denote by U_μ the logarithmic potential of the measure μ. We show that if $\alpha' = \beta'$ and if $\{(n+1)(U_{\alpha_n} - U_\alpha)\}$, $\{n(U_{\beta'_n} - U_{\beta'})\}$ uniformly have at most logarithmic growth at ∂D, then for every $f \in A^{-\infty}(D)$ and for the rational interpolants $r_{n,f}$ of degree n with poles at B_n interpolating to f at A_n, we have $r_{n,f} \to f$ in $A^{-\infty}(D)$, as $n \to \infty$.

Hyperbolic type metrics in Geometric function theory

by

Parisa Hariri
University of Turku, Finland

My talk is based on four papers [CHKV, HVW, HVZ, DHV], where metrics are central theme. As well-known, Geometric Function Theory is a field where metrics are recurrent: some examples are Euclidean, chordal and hyperbolic metrics. Here we study several other metrics defined on subdomains of $\mathbf{R}^\mathbf{n}$, $n \geq 2$. Examples include

- the triangular ratio metric
- the visual angle metric
- the distance ratio metric and its modification.

The first two of these metrics have been introduced and studied recently.

Some of the basic questions are:

- How are these metrics related to other metrics such as hyperbolic or quasihyperbolic metric?
- How are these metrics transformed under well-known classes of mappings such as Möbius transformations, quasiconformal maps, Lipschitz maps?

The answers will depend on the domains studied.

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Multistage-multivalue methods with inherent stability property for ordinary differential equations

by

G. Hojjati
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In this paper we deal with the construction of second derivative general linear methods [1, 2, 3] for the numerical solution of ordinary differential equations. To overcome the difficulties in the construction of the methods of high orders with desirable stability properties, we introduced some sufficient conditions which guarantee considered properties. Using the introduced strategies, we construct some methods of high orders and implement them in variable stepsize environment on some well-known stiff problems.

This presentation is based on a joint work with A. Abdi and S. A. Hosseini.

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On the numerical solution of nonlinear systems of delay Volterra integro-differential equations with constant delay

by

S. A. Hosseini
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Delay Volterra integro-differential equations (DVIDEs) with constant delay arise widely in the mathematical modeling of physical and biological phenomena. A practical example of this kind of equations is the modeling of the cohabitation of different biological species (see, e.g., [2]). In this study, we investigate the numerical solution of nonlinear systems of DVIDEs using a method based on the barycentric rational interpolatory quadrature rules [1, 3, 4]. Efficiency and accuracy of the proposed methods are confirmed by giving some numerical experiments.

This presentation is based on a joint work with A. Abdi

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Tightness results for infinite-slit limits of the chordal Loewner equation

by

Ikkei Hotta
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In [dMS16], the authors noted that the conformal mappings for a certain multiple SLE (Schramm-Loewner evolution) process for N simple curves in the upper half-plane $\mathbb{H} := \{z \in \mathbb{C} : \mathsf{Im}\, z > 0\}$ converges as $N \to \infty$. The deterministic limit has a simple description: The conformal mappings $f_t : \mathbb{H} \to \mathbb{H}$ satisfy the Loewner PDE

$$\frac{\partial f_t(z)}{\partial t} = -\frac{\partial f_t(z)}{\partial z} \cdot M_t(z), \quad f_0(z) = z \in \mathbb{H},$$

where M_t satisfies the complex Burgers equation

$$\frac{\partial M_t(z)}{\partial t} = -2\frac{\partial M_t(z)}{\partial z} \cdot M_t(z).$$

In several situations, partial differential equations of this type appear to describe the limit of N-particle systems.

In this talk, we consider the same multiple SLE measure for N curves connecting N points on $\mathbb{R}$ with ∞. We describe the growth of these curves by a Loewner equation with weights that correspond to the speed for these curves in the growth process, and we obtain an abstract differential equation for limit points as $N \to \infty$.

Furthermore, we see that an equation of a similar type also appears in the limit behavior of a Loewner equation describing the growth of trajectories of a certain quadratic differential.

This presentation is based on a joint work with Andrea Del Monaco and Sebastian Schleißinger [dMHS]**.**

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Linear differential equations with slowly growing solutions

by

Juha-Matti Huusko
Early Stage Researcher, University of Eastern Finland

This research concerns linear differential equations in the unit disc of the complex plane. In the higher order case the separation of zeros (of maximal multiplicity) of solutions is considered, while in the second order case slowly growing solutions in H^∞, BMOA and the Bloch space are discussed. A counterpart of the Hardy-Stein-Spencer formula for higher derivatives is proved, and then applied to study solutions in the Hardy spaces.

This presentation is based on a joint work with Janne Gröhn and Jouni Rättyä [1]**.**

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Meromorphic solutions of some algebraic difference equations in the complex plane

by

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In this talk, we are concerned with some difference equations in the complex plane. The theory of complex differential equations and the theory of complex discrete functional equations have been developed by giving impacts and influences each other with the remarkable developments of complex analysis.

Researches of algebraic differential equations and complex oscillation theory have been evolved by virtues of the Nevanlinna theory and the Wiman-Valiron theory. The considerations of the counterparts of these researches for discrete functional equations have required the construction of discrete versions of the value distribution theory of meromorphic functions. During in the last decade, the progress of difference analogues of the Nevanlinna theory have advanced. The difference analogues of the Wiman-Valiron theory were constructed and have been applied to built the counterparts of the theory of algebraic difference equations and linear differential equations in the complex plane, e.g., [1].

Concerning the interrelations between solutions of difference equations and solutions of differential equation, we have a method "continuous limit" which has been contributed to Painlevé analysis, e.g., [5]. Here, we apply continuous limits to give connections between solutions of certain classes of difference equations and solutions of the corresponding differential equations, [2], [3], [4]. Some examples are given.

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Holomorphic semicocycles in Banach spaces

by

Fiana Jacobzon
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We study holomorphic semicocycle in general Banach spaces. Semicocycles play an important role in the theory of dynamical systems and are closely connected to semigroups of weighed composition operators (for the holomorphic one-dimensional case, see [4]). In addition, we show the connection between semicocycles and extension operators for semigroups.

We introduce a semicocycle as a family of holomorphic mappings, whose values are bounded linear operators. We study boundedness, invertibility and differentiability of semicocycles. It turns out that some properties of semicocycles are, in a sense, similar to properties of semigroups of linear operators.

This presentation is based on a joint work with M. Elin, and H. Katriel

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Harmonic Bergman spaces and hypergeometric functions

by

Jiří Jahn

Silesian University in Opava

It is widely known that for the weighted Bergman space $L^2_{hol}(\Omega, w)$ of holomorphic and square-integrable functions on a smoothly bounded strictly pseudoconvex domain $\Omega \subset \mathbb{C}^n$, the Berezin transform

$$B_w f(x) = \frac{\langle f K_{\alpha,x}, K_{\alpha,x}\rangle}{\langle K_{\alpha,x}, K_{\alpha,x}\rangle} = \int_\Omega f(y) \frac{|K_\alpha(x,y)|^2}{K_\alpha(x,x)} w(y)^\alpha \, \mathrm{d}y,$$

where w is a suitably chosen weight, $K_{\alpha,y} := K_\alpha(\cdot, y)$ is the reproducing kernel of $L^2_{hol}(\Omega, w)$, and f is a bounded smooth function on Ω, has the following asymptotic behavior:

$$B_\alpha f \sim \sum_{j=0}^{\infty} \frac{Q_j f}{\alpha^j}, \quad \text{as } \alpha \longrightarrow \infty, \ \alpha \in \mathbb{Z}, \tag{1}$$

In the talk we discuss some recent results and several open problems connected with the asymptotic behaviour of the type (1) of the analogous Berezin transform defined in the context of the harmonic (rather than holomorphic) Bergman spaces and its possible connection to multivariable hypergeometric functions.

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A new numerical differentiation formula to approximate the fractional differential equations

by

Mohammad Javidi
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A novel computationally effective numerical algorithm is proposed for solving fractional differential equations where the fractional derivative is considered in the Caputo sense. The detailed error analysis is presented and it is analytically proven that the proposed algorithms are of order four[2]. The stability of the algorithms is rigorously established and the stability region is also achieved [3, 4]. Numerical examples are provided to illustrate the efficiency and applicability of the novel algorithm.

This presentation is based on a joint work with Mohammad Shahbazi Asl.

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On the zeros of Meixner and Meixner-Pollaczek polynomials

by

AS Jooste
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Josef Meixner was a German physicist and the Meixner polynomials, as well as the Meixner-Pollaczek polynomials, that are considered to be Meixner polynomials with an imaginary argument, were introduced by him in 1934. These polynomials lie on the ${}_2F_1$ plane of the Askey scheme of hypergeometric orthogonal polynomials. We discuss the properties of the zeros of these polynomials as well as the quasi-orthogonality of these systems of polynomials, using a characterisation of quasi-orthogonality due to Riesz. Of particular interest are the Meixner-Pollaczek polynomials, whose linear combinations only exhibit quasi-orthogonality of even order.

Muckenhoupt weights and Lindelöf theorem for harmonic mappings

by

David Kalaj
University of Montenegro, Montenegro

We extend the result of Lavrentiev which asserts that the harmonic measure and the arc-length measure area ∞ equivalent in a chord-arc Jordan domain. By using this result we extend the classical result of Lindelöf to the class of quasiconformal (q.c.) harmonic mappings by proving the following assertion. Assume that f is a quasiconformal harmonic mapping of the unit disk $\mathbf{U}$ onto a Jordan domain. Then the function $A(z) = \arg(\partial_\varphi(f(z))/z)$ where $z = re^{i\varphi}$, is well-defined and smooth in $\mathbf{U}^* = \{z : 0 < |z| < 1\}$ and has a continuous extension to the boundary of the unit disk if and only if the image domain has C^1 boundary.

Rational Bernstein- and Markov-type inequalities

by

Sergei Kalmykov
Shanghai Jiao Tong University, China

In this talk we consider asymptotically sharp extensions of the classical Bernstein and Markov inequalities for rational functions on Jordan arcs and curves (see [1]). The asymptotically sharp constants there can be expressed via the normal derivatives of certain Green's functions with poles at the poles of the rational functions in question. In the proofs key roles are played by Borwein-Erdélyi inequality on the unit circle, Gonchar-Grigorjan type estimate of the norm of holomorphic part of meromorphic functions, Totik's construction of fast decreasing polynomials, and conformal mappings.

This presentation is based on a joint work with Béla Nagy and Vilmos Totik

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Logarithmic convexity and concavity of generalized hypergeometric functions with respect to multiple parameter shifts

by

Dmitrii Karp
Far Eastern Federal University, Russian Federation

Given a vector $\mathbf{a}$ and a scalar μ define $\mathbf{a}+\mu$ as $(a_1+\mu, a_2+\mu, \ldots, a_p+\mu)$. In the talk we discuss the logarithmic convexity and concavity of the function

$$\mu \to f(\mu; x) = \phi(\mu)_p F_q \left(\begin{matrix} \mathbf{a}_1, \mathbf{a}_2 + \mu \\ \mathbf{b}_1, \mathbf{b}_2 + \mu \end{matrix} \middle| x \right),$$

where ${}_pF_q$ is the generalized hypergeometric function and

$$\phi(\mu) \in \left\{ 1, \prod_{a_i \in \mathbf{a}_2} \Gamma(a_i + \mu), \frac{1}{\prod_{b_i \in \mathbf{b}_2} \Gamma(b_i + \mu)}, \frac{\prod_{a_i \in \mathbf{a}_2} \Gamma(a_i + \mu)}{\prod_{b_i \in \mathbf{b}_2} \Gamma(b_i + \mu)} \right\}.$$

Here $\mathbf{a}_i$, $\mathbf{b}_i$ are real vectors of lengthes p_i and q_i, respectively, with $p = p_1 + p_2$, $q = q_1 + q_2$. We also consider the power series coefficients of the generalized Turánian

$$f(\mu + \alpha; x) f(\mu + \beta; x) - f(\mu; x) f(\mu + \alpha + \beta; x)$$

and connection to Laguerre-Pólya class of entire functions. We further mention generalizations to series with generic terms involving product ratios of gamma functions and certain conjectures for combinatorial polynomials arising in these investigations.

The results presented in the talk have been obtained jointly with S.I. Kalmykov. Financial support of the Russian Science Foundation under project 14-11-00022 is acknowledged.

This presentation is based on a joint work with S.I. Kalmykov

On Dynamical and Parametric Zalcman Functions

by

Tomoki Kawahira
Tokyo Institute of Technology

We apply Zalcman's lemma [Za] to

- dynamics of rational maps on the Riemann sphere of degree two or more; and
- the bifurcation locus of a family of rational maps.

Then we have families of non-constant meromorphic functions that we call the *dynamical* and *parametric Zalcman functions* (Cf. [St]). In this talk, we present some basic properties of these families and give a simple proof of the local similarities between the Julia sets, Mandelbrot set, and the tricorn (Tan Lei, J.Rivera-Letelier, and [Ka]) by using Zalcman functions.

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Synchronization and anti-synchronization of fractional order chaotic systems and their application in secure communication

by

H. Kheiri

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In this paper, we consider two fractional order chaotic systems and study the qualitative behavior of them. We introduce some effective methods for synchronization and anti-synchronization of fractional order chaotic systems and apply them to two considered systems in Caputo sense. The asymptotic stability of error system is proved using Lyapunov stability of fractional order systems. Moreover, synchronization and anti-synchronization result is applied to secure communication. For verification of theory topics, numerical simulations are given.

This presentation is based on a joint work with B. Naderi

Constructive proof of the Weierstrass Theorem for weighted functions on unbounded intervals

by

Theodore Kilgore
Auburn University, United States

The following two results will be presented.

Under the condition that the weight function W is positive and continuous on $[0, \infty)$ and satisfies $\frac{-\ln W(x)}{x^{\alpha}} \to \infty$ as $x \to \infty$ for some $\alpha > 1$, then the approximation theorem of Weierstrass is shown to hold for $f(x) \in C_W^0[0, \infty)$, the set of continuous functions for which $W(x)f(x) \to 0$ as $x \to \infty$.

Similarly, if W defined on $(-\infty, \infty)$ and is positive and continuous and satisfies $\frac{-\ln W(x)}{x^{2\alpha}} \to \infty$ as $x \to \pm\infty$ for some $\alpha > 1$, then the approximation theorem of Weierstrass is shown to hold for $f(x) \in C_W^0(-\infty, \infty)$, the set of continuous functions for which $W(x)f(x) \to 0$ as $x \to \pm\infty$.

The proofs of the above two results will be based upon a generalization by Chlodowsky of the Bernstein polynomial operators.

These results are presented in memory of Katalin Balázs 08/13/1949 - 09/07/2016, who was for 26 years my mathematical collaborator and my very devoted and very beloved wife.

Approximation and Shape Preserving Properties of the Jain Operator of Max-Product Kind

by

Sevilay Kirci Serenbay
Başkent University, Faculty of Education, Department of Mathematics and Science Education, Ankara, Turkey

In the classical approximation theory the approximating polynomials are exclusively linear operators. In the recent years Bede et. al. ([1]) defined nonlinear operators which are formed by using maximum ($\bigvee$) and product (.) operations. They studied the approximation properties of these new operators and showed that they have similar properties as the ones obtained by classical approximation process. In the light of these studies, in this paper we introduce a new operator of max product kind. We present the Jain operators of max-product kind and we give an estimate for the error of approximation using modulus of continuity.

This presentation is based on a joint work with Özge Dalmanoğlu.

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C-symmetric, skew–symmetric operators, and reflexivity

by

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University of Agriculture, Krakow

Let $\mathcal{H}$ be a complex Hilbert space with an inner product $\langle .,. \rangle$, and let $B(\mathcal{H})$ be the Banach algebra of all bounded linear operators on $\mathcal{H}$. By F_k we denote the set of all operators of rank at most k. A weak* closed subspace $\mathcal{S} \subset B(\mathcal{H})$ is *k-reflexive* if rank-k operators are linearly dense in $\mathcal{S}_\perp = \{t\text{–a trace -class operator} : tr(St) = 0 \text{ for all } S \in \mathcal{S}\}$. A subspace $\mathcal{S}$ is called *k-hyperreflexive* if there is a constant $c > 0$ such that

$$\operatorname{dist}(T, \mathcal{S}) \leq c \cdot \sup\{|tr(Tt)| : t \in F_k \cap \mathcal{S}_\perp, \|t\|_1 \leq 1\},$$

for all $T \in B(\mathcal{H})$. Note that $\operatorname{dist}(T, \mathcal{S})$ is the infimum distance.

Recall that C is a *conjugation* on $\mathcal{H}$ if $C : \mathcal{H} \longrightarrow \mathcal{H}$ is an antilinear, isometric involution, i.e., $\langle Cx, Cy \rangle = \langle y, x \rangle$ for all $x, y \in \mathcal{H}$ and $C^2 = I$. An operator T in $B(\mathcal{H})$ is said to be *C–symmetric* if $CTC = T^*$ and T is said to be *skew–C symmetric* if $CTC = -T^*$.

During the talk we will discuss reflexivity and hyperreflexivity of the space of all C–symmetric or skew–C symmetric operators.

This presentation is based on a joint work with Chafiq Benhida and Marek Ptak.

Zero sequences and factorization for weighted Bergman spaces

by

Taneli Korhonen
University of Eastern Finland

The first comprehensive study of zero sets of functions in Bergman spaces of the unit disc was done by Horowitz in 1974. He then used these results to prove a well-known factorization theorem for Bergman spaces. Later, Luecking gave a more delicate characterization of the zero sequences in the setting of mixed norm spaces. It turns out that Luecking's results can be generalized to the case of weighted Bergman spaces and used in the study of factorization results to improve certain constants in the norm estimates. In this talk, I give a characterization of the zero sequences for the weighted Bergman space A^p_ω induced by either a radial weight ω admitting a certain doubling property or a non-radial Bekollé-Bonami type weight. Accurate results obtained en route to this characterization are then used to generalize Horowitz's factorization result for functions in A^p_ω. The utility of the obtained factorization is illustrated by applications to integration and composition operators as well as to small Hankel operator induced by a conjugate analytic symbol.

This presentation is based on a joint work with Jouni Rättyä

Delay differential Painlevé equations and Nevanlinna theory

by

Risto Korhonen
University of Eastern Finland

Necessary conditions are obtained for certain types of rational delay differential equations to admit a non-rational meromorphic solution of hyper-order less than one. The equations obtained include delay Painlevé equations and equations solved by elliptic functions.

This presentation is based on a joint work with Rod Halburd.

On deviations and spreads of meromorphic minimal surfaces

by

Arnold Kowalski
West Pomeranian University of Technology, School of Mathematics, Poland

In 1969 E.F. Beckenbach and collaborators generalized the Nevanlinna theory by introducing theory of meromorphic minimal surfaces (m.m.s).
The main goal of this talk is to give sharp estimates of spread of a m.m.s. and an estimate of the sum of magnitudes od deviations of m.m.s. of finite lower order.
The spread of a m.m.s. was defined by V.P. Petrenko in 1981. We present sharp estimations of spread in terms of Nevanlinna's defect, magnitude of deviation and the number of separated point of the norm of m.m.s. We also give examples showing that the estimates are sharp.

This presentation is based on a joint work with Iwan Marczenko

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More accurate Jensen-type inequalities obtained via linear interpolation and applications

by

Mario Krnić
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In this talk we present a general method for improving Jensen-type inequalities for convex and, even more generally, for piecewise convex functions. Our main result relies on linear interpolation of a convex function. As a consequence, we obtain improvements of some recently established Young-type inequalities in both real and matrix case.

This presentation is based on a joint work with Daeshik Choi and Josip Pečarić.

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Generalizations and refinements of Opial type inequalities

by

Kristina Krulić Himmelreich
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This talk deals with Opial inequality and its famous generalizations, extensions and refinements, i.e. with Opial-type inequalities. In 1960, Opial proved the following inequality (see [7]):
Let $f \in C^1[0,h]$ be such that $f(0) = f(h) = 0$ and $f(x) > 0$ for $x \in (0,h)$. Then

$$\int_0^h |f(x)f'(x)|dx \leq \frac{h}{4}\int_0^h [f'(x)]^2 dx,$$

where $h/4$ is the best possible. This inequality has been generalized and extended in several directions. We obtain general Opial-type inequalities for measurable functions and for the quotient of functions. Application to numerous symmetric functions gives results that involve Green's functions, Lidstone's series and Hermite's interpolating polynomials.

This presentation is based on a joint work with Ana Barbir and Josip Pečarić.

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Extended Schwarz Lemma, complex geodesics and bounds for coefficients of holomorphic functions

by

Samuel L. Krushkal
Bar-Ilan University, Israel

We provide some extensions of the classical Schwarz lemma which are applied to estimating the coefficient functionals on various classes of bounded holomorphic functions.

Let $H(\mathbf{D}, G) \subset H^\infty$ be the collection of holomorphic maps $f(z) = c_0 + c_1 z + \dots$ of the unit disk into a multiply connected domain G in $\mathbf{D}$ bounded by the unit circle S^1 and at most countably many continua (may be degenerated to a point) which do not accumulate to S^1. We consider on such general classes the perturbed coefficient functionals of the form

$$J(f) = c_n + F(c_{m_1}, \dots, c_{m_s}), \quad n > 1,$$

where $c_j = c_j(f)$, $1 \le m_j < n$ and F is a holomorphic functions on an appropriate domain in $\mathbf{C}^s$ such that F, ∂F vanish at the origin $\mathbf{0}$, and prove that for any such a functional there exist a value $k_0 = k_0(G, J) > 0$ such that for all $f \in H(\mathbf{D}, G)$ with $\|f\|_\infty \le k_0$ we have the sharp estimate $|J(f)| \le |c_1(\kappa_G)|$, where κ_G is a universal covering map $\mathbf{D} \to G$ with maximal $|f'(0)|$.

In the case when G is a circular ring $\{\rho < |z| < 1\}$ $(\rho \ge 0)$ and $J(f)$ is homogeneous, the above distorion estimate is valid for $\|f\|_\infty \le 1$. This gives, in particular, a proof of the well-known Krzyz conjecture and its generalizations.

Our approach involves the complex geodesics and quasiconformal theory.

A Leibniz-type rule for random variables

by

Zoltán Léka
Royal Holloway, University of London

For differentiable functions on the real line the Leibniz rule and Hölder's inequality provides us with a simple way to have various estimates of the L^p norms of derivatives of products. Recently, Leibniz-type rules have appeared in the theory of non-commutative metric spaces established by M. Rieffel. Furthermore, from the perspective of differential operators, the Kato–Ponce inequality or the fractional Leibniz rule related to the Laplace operator have intensively been studied in PDEs as well.

In this talk we shall present a Leibniz-type inequality for functions defined on probability spaces. Namely, we prove that if $(\Omega, \mathcal{F}, \mu)$ is a probability space then for any real $f, g \in L^\infty(\Omega, \mu)$ one has

$$\|fg - \mathbb{E}(fg)\|_r \leq \|f\|_{p_1} \|g - \mathbb{E}g\|_{q_1} + \|g\|_{p_2} \|f - \mathbb{E}f\|_{q_2},$$

where $1 \leq r, p_1, p_2, q_1, q_2 \leq \infty$ and $\frac{1}{r} = \frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{p_2} + \frac{1}{q_2}$.

The proof is based on convex analysis and majorization theory.

Hayman's List - an update

by

Eleanor F. Lingham
Sheffield Hallam University

In 1967, Walter K. Hayman published *Research Problems in Function Theory*, a list of 140 research problems in seven areas of function theory, including meromorphic functions, Nevalinna theory and functions in the unit disc. These classical problems were contributed by Hayman, and other mathematicians including Erdös, Turán and Collingwood, and became a focus of function theory research. Over the following decades, several updates have been published, with each tracking the development of the conjectures, and adding new problems. *Hayman's List* now stands at 515 problems in nine areas of complex analysis.

This year, for the fiftieth anniversary of its publication, Walter Hayman and Eleanor Lingham are updating this work. In this short talk, I will share some of the problems, look at developments, and ask for your help in finding out which conjectures have been resolved.

This presentation is based on a joint work with Walter K. Hayman.

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Almost all continued fractions converge

by

Lisa Lorentzen
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Let $\mathbf{K}(a_n/b_n)$ be a continued fraction with elements (a_n, b_n) picked randomly and independently from $\Omega := (\mathbb{C} \setminus \{0\}) \times \mathbb{C}$ according to some given probability distribution μ on Ω. This induces a probability distribution ψ on the space $\mathbb{K}$ of continued fractions $\mathbf{K}(a_n/b_n)$. We prove that the set of all divergent continued fractions in $\mathbb{K}$ has ψ-measure 0 under very mild conditions on μ. We further claim that similar results also hold

- in the periodic case where $\{(a_{np+k}, b_{np+k})\}_{n=1}^{\infty}$ is picked randomly and independently from Ω according to a distribution depending on k for $k = 1, 2, \ldots, p$
- in the vector case where $(\mathbf{a}_n,\, b_n) \in \mathbb{C}^N \times \mathbb{C}$ for some $N \in \mathbb{N}$
- in the case where $(a_n,\, b_n)$ are pairs from the Clifford group,
- for $\{\mathcal{T}_n\}$ where $\mathcal{T}_n = \tau_1 \circ \cdots \circ \tau_n$ with independent μ-random $\{\tau_n\}$ from the space of Möbius transformations.

Planar Harmonic Mappings of Bounded Boundary Rotation

by

Abdallah K. Lyzzaik
National Council for Scientific Research, CNRS, Beirut, Lebanon

The purpose of this talk is to give a brief introduction about planar harmonic mappings and to investigate the valency of some classes of planar harmonic mappings of the open unit disc $\mathbb{D}$. The results that will be presented were motivated by the earlier works [*Close-to-convexity criteria for planar harmonic mappings*, Complex Analysis Oper. Theory, 5 (2011), 767-774] and [*A sufficient condition for p-valency of harmonic mappings*, Complex Var. Elliptic Equ. 59 (2014), 1214-1222]. The main result in this regard is a valency criterion for planar harmonic mappings of bounded boundary rotation of $\mathbb{D}$.

Manysheeted Variants of Polya-Bernstein and Borel Theorems for Entire Functions of Order $\rho \neq 1$ and Some Applications

by

Lev Maergoiz
Siberian Federal University, Krasnoyarsk, Russia

It is well known the G. Polya theorem on the relationship between indicator and conjugate diagrams of an entire function of exponential type [1, Lecture 9]. It is represented a variant of this theorem, based on the V. Bernstein construction [2] of a many-sheeted indicator diagram of an entire function of order $\rho \neq 1$ and normal type. It is given a description of the analytic continuation domain for Puiseux series, converging in a neighbourhood of the point at infinity. As a consequence it was found a summation domain of a "principal" Puiseux series by the Borel method (a many-sheeted "Borel polygon") [3, ch. 8], [4, ch. 3].

These assertions are applied for a description of the analytic continuation domain of Puiseux series – expansions of many-sheeted functions (for an example, conversions of rational functions).

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Spherical derivative and closeness of a-points

by

Marina Makhmutova
Sultan Qaboos University

A meromorphic in $\mathbb{C}$ function f is called p-Yosida function, $p \geq 1$, if

$$\sup_{z \in \mathbb{C}} |z|^{2-p} \frac{|f'(z)|}{1+|f(z)|^2} < \infty.$$

According to Montel's principle of normality: f is a p-Yosida function, $p > 1$, if and only if for any sequence $\{a_n\} \subset \mathbb{C}$, $\lim_{n\to\infty} a_n = \infty$, the family $\{f(a_n + z|a_n|^{2-p})\}$ is normal in $\mathbb{C}$. Case $p = 1$ is special and it coincides with Julia exceptional functions. In that case the family $\{f(a_n z)\}$ is normal in $\mathbb{C} \setminus \{0\}$.

In [1] p-Yosida functions are described in terms of closeness of a-points. Using closeness property we can show that product of p-Yosida functions is not a p-Yosida function.

This presentation is based on a joint work with Shamil Makhmutov

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Boundary behaviour of Dirichlet series and applications

by

Myrto Manolaki
University of South Florida

This talk is concerned with the boundary behaviour of holomorphic functions representable as absolutely convergent Dirichlet series in a half-plane, in connection with the convergence properties of partial sums of the Dirichlet series on the boundary. We will discuss some main results and we will see how they can be applied to obtain new information about universal Dirichlet series and universal Taylor series.

This presentation is based on a joint work with Stephen Gardiner.

A harmonic maps approach to fluid flows

by

María J. Martín
University of Eastern Finland

Several publications in the applied mathematics, engineering and physics research literature exploited a remarkable feature shared by some celebrated explicit solutions to the two-dimensional incompressible Euler equations in Lagrangian variables, namely that in all of them the labelling map is harmonic at all times.

Recently, A. Aleman and A. Constantin proposed a complex analysis approach aimed at classifying all such flows. While new explicit solutions were obtained, the exhaustion of all possibilities was reduced to an explicit nonlinear ordinary differential system in $\mathbb{C}^4$. Solving this system in full generality proved elusive so far.

We propose a different approach, based on ideas from the theory of harmonic mappings, that provides a complete solution to the problem of classifying all two-dimensional ideal fluid flows with harmonic Lagrangian labelling maps; thus, we explicitly provide all solutions (with the specified structural property) to the incompressible two-dimensional Euler equations.

This presentation is based on a joint work with Olivia Constantin.

Univalence criteria for local homeomorphisms with application to planar harmonic mappings

by

Andrzej Michalski
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Let Ω be a subset of the complex plane $\mathbb{C}$ such that Ω is a finite union of domains convex in the horizontal direction. We study topological properties of the sets Ω and Ψ, where $\Psi := q(\Omega)$ and $q : \Omega \to \mathbb{C}$ is a local homeomorphism such that $\mathrm{Im}\{q(z)\} = \mathrm{Im}\{z\}$ for each $z \in \Omega$. Under some additional conditions on Ω and Ψ, q is proved to be a global homeomorphism. In particular, if Ω is a starlike domain and Ψ is simply connected, then q is one-to-one in Ω. By applying these results to planar harmonic mappings we obtain some generalisations of the shear construction theorem due to Clunie and Sheil-Small.

This presentation is based on a joint work with Małgorzata Michalska.

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Asymptotic expansions of some n-variable means

by

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Series $\sum_{n=0}^{\infty} a_n x^{-n}$ is said to be an asymptotic expansion of a function $f(x)$ as $x \to \infty$ if for each $N \in \mathbf{N}$

$$f(x) = \sum_{n=0}^{N} a_n x^{-n} + o(x^{-N}).$$

In our recent papers asymptotic expansions of many bivariate classical and parameter means were found, using the explicit formulas for observed means. The coefficients in the asymptotic expansions were useful in analysis of considered means. After having these expansions calculated it was natural to extend this principle on n-variable means.

Here we discuss relations between some n-variable classical means, using technique of asymptotic expansions. For an n-tuple of positive real numbers $\mathbf{a} = (a_1, a_2, \ldots, a_n)$ we observe the behavior of the power means with respect to the shifted variable $x\mathbf{e} + \mathbf{a}$, where $\mathbf{e} = (1, 1, \ldots, 1)$, when x approaches infinity. General expansions of this means were known in terms of Bell polynomials while we provided simple recursive algorithms. Coefficients $c_k(\mathbf{a})$ in the expansion

$$M_r(x\mathbf{e} + \mathbf{a}) = x \sum_{k=0}^{\infty} c_k(\mathbf{a}) x^{-k}$$

were obtained in terms of power mean $M_r(\mathbf{a})$ with integer order r.

Such obtained coefficients were used in the analysis of some inequalities which include the first asymptotic term. Based on the asymptotic expansions of the considered means we introduce a notion of asymptotic inequality which turns out to be a necessary condition for the proper inequality.

This presentation is based on a joint work with prof. Neven Elezović

Harmonic Faber Polynomials and Harmonic Faber Expansions

by

Marvin Müller
Universität Trier

The Faber polynomials F_n and series expansions in F_n play an important role in the theory of polynomial approximation in the uniform norm on compact plane sets. We consider series expansions in F_n and the complex conjugate $\overline{F_n}$ and give conditions for their convergence. In the special case of the unit disk, these expansions correspond to Fourier series. In some good-natured situations, this leads to harmonic polynomials which approximate solutions of Dirichlet problems. For concrete examples, we numerically calculate the coefficients of these polynomials and produce graphic visualisations on the base of our numerical results.

Some results on open-up with overview and applications

by

Béla Nagy
MTA-SZTE Analysis and Stochastics Research Group

In this talk rational functions with prescribed critical values are investigated. Existence is established in two different ways, using Joukowskii mapping, and using Riemann surfaces. In this latter case minimality is observed, that is, the degrees of the numerator and denominator are as small as possible. We also plan to outline a numerical method on approximating the minimal degree rational function. This work is in progress.

In the overview part, several results are going to be mentioned which are related: Poincare, Joukowskii, Myrberg (1922), Thom (1965), Widom (1969), Mycielski (1970), Mednykh (1984), Goldberg (1991), Beardon -Crane-Ng (2002), Seppala (2004), Hidalgo-Seppala (2011).

As an application, an open-up method of finite systems of Jordan arcs on the complex plane is discussed, paying attention to smoothness. Finally, this open-up method is used to show sharp Bernstein type inequality on arcs.

This presentation is based on joint works with Olivier Sète and Sergei Kalmykov.

Hyperfunction method for numerical integration and Fredholm integral equations of the second kind

by

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In this presentation, we propose a numerical method for one-dimensional integration and Fredholm integral equation of the second kind based on hyperfunction theory, which is a generalized function theory based on complex function theory [3].

We consider the computation of the integral of an analytic function. According to hyperfunction theory, the desired integral is given by a complex loop integral [1]. In our method, we compute the integral by evaluating the complex loop integral using the trapezoidal rule with uniform mesh. A theoretical error estimate shows that our method converges geometrically, and numerical examples show that our method is efficient especially for integrals with strong end-point singularities [1].

In addition, we apply our numerical integration method to Fredholm integral equations of the second kind. Figure 1 shows the errors of our method and the DE-Sinc collocation method [2] applied to the integral equation

$$f(x) + \int_{-1}^{1} (x-\xi) f(\xi) \mathrm{d}\xi = \mathrm{e}^{x} + 2\{(\sinh 1)x - \mathrm{e}^{-1}\},$$

whose exact solution is $f(x) = \mathrm{e}^{x}$. This shows the effectiveness of our method.

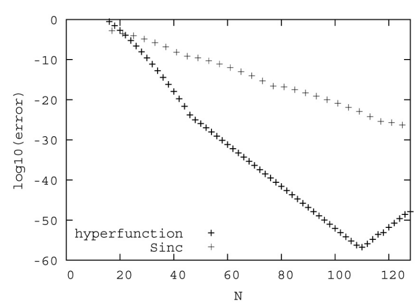

Figure 1: The errors of our method and the DE-Sinc method, where N is the number of nodes.

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On subordination associated with certain classes of analytic functions define by q-derivative operator

by

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In this paper, we discuss subordination results for certain classes of functions analytic in the unit disk and define by q-derivative operator. We also, provide lower bounds for the real part of functions belonging to the classes.

Keywords: Analytic functions, subordination, q-derivative operator.

Quasiconformality of harmonic mappings in the unit disk

by

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and
The State School of Higher Education in Chełm, Poland

We present results involving the following quasiconformality property

$$\sup_{z\in\mathbb{D}}\left|\frac{G'(z)}{H'(z)}\right| < 1$$

of an injective and sense-preserving harmonic mapping $F = H + \overline{G}$ in the unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$, where H and G are holomorphic functions in $\mathbb{D}$. In particular, we discuss the Lipschitz type conditions for a quasiconformal harmonic mapping $F : \mathbb{D} \to \mathbb{C}$ under additional assumptions like the convexity of $F(\mathbb{D})$ or $H(\mathbb{D})$.

This presentation is based on a joint work with Ken-ichi Sakan and Jian-Feng Zhu.

Pluripotential numerics

by

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We introduce numerical methods for the approximation of the main (global) quantities in Pluripotential Theory as the *extremal plurisubharmonic function* V_E^* of a compact $\mathcal{L}$-regular set $E \in \mathbb{C}^n$, its *transfinite diameter* $\delta(E)$, and the *pluripotential equilibrium measure* $\mu_E := (\mathrm{dd}^{\mathrm{c}}\, V_E^*)^n$.

The methods rely on the computation of a *polynomial mesh* for E and numerical orthonormalization of a suitable basis of polynomials. We prove the convergence of the approximation of $\delta(E)$ and the uniform convergence of our approximation to V_E^* on all $\mathbb{C}^n$; the convergence of the proposed approximation to μ_E follows. Our algorithms are based on the properties of polynomial meshes and Bernstein Markov measures.

Numerical tests are presented for some simple cases with $E \subset \mathbb{R}^2$ to illustrate the performances of the proposed methods.

On the solution of time fractional mobile/immobile equation using spectral collocation method

by

H. Pourbashash

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Spectral Methods are the family of numerical methods for solving engineering problem which are well-known for powerful deep-seated theorem and high-order accuracy in application. Spectral collocation methods solve the problem in physical space against spectral Galerkin methods that implemented in frequency space. The mobile/immobile equation is an time fractional partial differential equation which arise from hydrology. The fractional derivative of equation is described in the Caputo sense. We use an spectral collocation method based on Legendre polynomials for the solution of mobile/immobile equation. In this method, the unknown solution to the differential equation is expanded as global polynomial interpolants based on some suitable points which called as Legendre-Gauss-Lobatto points . The aim of this paper is to show that the spectral method based on the Legendre polynomial is also suitable for the treatment of the fractional partial differential equations. Numerical examples confirm the high accuracy of the proposed scheme.

Radius of Starlikeness and Hardy Space of Mittag-Leffler functions

by

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A special function of growing importance is the generalized Mittag-Leffler function defined by [2]

$$E_{\lambda,\mu}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\lambda n+\mu)}\quad(\lambda,\mu,z\in \mathrm{C},\,\Re(\lambda)>0,\,\Re(\mu)>0),\tag{1}$$

for complex argument $z\in \mathrm{C}$ and parameters $\lambda,\mu\in \mathrm{C}$ with $\Re(\lambda)>0$. The Mittag-Leffler function (1) is an entire functions of order $\rho=1/\lambda$ and type $\sigma=1$ [1, Corollary 1.2]. The Mittag-Leffler function is a generalization of the exponential function, to which it reduces for $\lambda=\mu=1$, $E_{1,1}(z)=\exp(z)$. Despite the fact that $E_{\lambda,\mu}$ was introduced roughly 110 years ago, its mapping properties in the complex plane are largely unknown.

We consider the following two normalization of the Mittag-Leffler function $E_{\lambda,\mu}(z)$:

$$\mathrm{E}_{\lambda,\mu}(z)=\Gamma(\mu)zE_{\lambda,\mu}(z)=z+\sum_{n=2}^{\infty}\frac{\Gamma(\mu)}{\Gamma(\lambda(n-1)+\mu)}z^n\tag{2}$$

$(\lambda>0,\mu>0,z\in \mathrm{D})$ and

$$\mathcal{E}_{\lambda,\mu}(z)=\left(\Gamma(\mu)\,z^{\mu}E_{\lambda,\mu}(z)\right)^{\frac{1}{\mu}}=z+\frac{\Gamma(\mu)}{\mu\,\Gamma(\lambda+\mu)}z^2+\cdots\tag{3}$$

$(\lambda>0,\mu>0,z\in \mathrm{D})$. Note that $\mathcal{E}_{\lambda,\mu}(z)=\exp\left\{\frac{1}{\mu}\operatorname{Log}\left(\Gamma(\mu)z^{\mu}E_{\lambda,\mu}(z)\right)\right\}$, where Log represents the principal branch of logarithm. Whilst formula (2) and (3) holds for complex-valued λ, μ, however, we shall restrict our attention to the case of $\lambda>0,\ \mu>0$.

In the present talk, we discuss a sufficient condition for the function $\mathrm{E}_{\lambda,\mu}$ to be starlike function of order α. Also, we discuss radius of starlikeness of order α for the functions $\mathrm{E}_{\lambda,\mu}$ and $\mathcal{E}_{\lambda,\mu}$. Further, we discuss results so that $\mathrm{E}_{\lambda,\mu}$ belongs to the Hardy spaces $\mathcal{H}^p$ and $\mathcal{H}^{\infty}$.

This presentation is based on a joint work with Suhananda Maharana

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Applications of integral representations of generalized hypergeometric functions

by

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Let us fix some notation and terminology first. As usual $\Gamma(z)$ stands for Euler's gamma function and $(a)_n = \Gamma(a+n)/\Gamma(a)$ denotes the rising factorial (or Pochhammer's symbol). Further, we will follow the standard definition of the generalized hypergeometric function ${}_pF_q$ as the sum of the series

$$ {}_pF_q\left(\begin{matrix}\mathbf{a}\\ \mathbf{b}\end{matrix}\middle|\, z\right) = {}_pF_q\left(\mathbf{a};\mathbf{b};z\right) = \sum_{n=0}^{\infty}\frac{(a_1)_n(a_2)_n\cdots(a_p)_n}{(b_1)_n(b_2)_n\cdots(b_q)_n n!}z^n $$

if $p \le q$, $z \in \mathbf{C}$. If $p = q+1$ the above series has unit radius of convergence and ${}_pF_q(z)$ is defined as analytic continuation of its sum to $\mathbf{C}\setminus[1,\infty)$. Here $\mathbf{a} = (a_1,\ldots,a_p)$, $\mathbf{b} = (b_1,\ldots,b_q)$ are (generally complex) parameter vectors, such that $-b_j \notin \mathbf{N}_0$, $j = 1,\ldots,q$.

In our previous work [1] we found conditions that need to be imposed on the parameters of the generalized hypergeometric function in order to ensure that it was completely monotonic or Stieltjes function. In this paper we collect a number of consequences of these properties. In particular, we find new integral representations for the generalized hypergeometric functions, evaluate a number of integrals of products of hypergeometric functions, compute the jump of the the generalized hypergeometric function over the branch cut $[1,\infty)$. It is a straightforward consequence of a result due to Thale [2] observed by us that the generalized hypergeometric function ${}_{p+1}F_p(z)$ is univalent in $\Re(z) < 1$ when it belongs to the Stieltjes class. Univalent functions satisfy a number of distortion theorems. In this way we establish new inequalities for the generalized hypergeometric function in the half plane $\Re z < 1$.

This work has been supported by the Russian Science Foundation under project 14-11-00022.

This presentation is based on a joint work with Dmitrii Karp.

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Asymmetric truncated Toeplitz operators and conjugations

by

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Let H^2 be the Hardy space on the unit disc, identified as usual with a subspace of L^2 on the unit circle. With any nonconstant inner function θ we associate the model space K^2_θ, defined by $K^2_\theta = H^2 \ominus \theta H^2$. In this space we can define the conjugation (antilinear, isometric, involution) $C_\theta \colon K^2_\theta \to K^2_\theta$ by $C_\theta f(z) = \theta \overline{z f(z)}$.

Let us consider two nonconstant inner functions α and θ such that α divides θ. For a given function $\varphi \in L^2$ we can define an asymmetric truncated Toeplitz operator $A_\varphi \colon K^2_\theta \to K^2_\alpha$ by $A_\varphi f = P_\alpha(\varphi f)$, where $P_\alpha \colon L^2 \to K^2_\alpha$ is the orthogonal projection. The relation between bounded asymmetric truncated Toeplitz operators with L^2 symbols and conjugations C_θ, C_α will be investigated. The relations are different to symmetric case $\theta = \alpha$.

This presentation is based on a joint work with C. Câmara and K. Kliś-Garlicka

Conformal Modulus and Numerical Conformal Mappings

by

Antti Rasila
Aalto University

Conformal moduli of rings and quadrilaterals are frequently applied in the geometric function theory, and these quantities are also useful in certain applications in numerical analysis. We recall the basic properties of the conformal modulus, and its role in the theory of harmonic and quasiconformal mappings and consider numerical computation of conformal moduli [4, 5, 6].

Besides approximating the conformal modulus, the harmonic potential function can also be used for finding the canonical conformal mapping by using the harmonic conjugate method first introduced in [3]. These techniques can be used for numerical computation of conformal mappings of simply and doubly connected domains [3], and there exists an interesting generalization of the algorithm for multiply connected domains [2]. We also discuss recent applications of numerical conformal mappings on modelling and optimization of Cellular Networks [1].

This presentation is based on a joint work with D. González G., H. Hakula, J. Hämäläinen, T. Quach and M. Vuorinen.

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Weighted Ostrowski and Grüss type inequalites with applications

by

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The goal of this talk is to present several weighted Ostrowski and Grüss type inequalites for continuous functions with one point of nondifferentiability by using some bounds for the Chebyshev functional. Also, some Grüss type inequalites for functions with absolutely continous n-th derivative will be presented. As applications, some new estimates for the error in certain numerical integration rules will be obtained.

This presentation is based on a joint work with Josip Pečarić.

Universal subleading asymptotics of planar orthogonal polynomials

by

Roman Riser
Holon Institute of Technology, Israel

We consider the monic orthogonal polynomial p_n of degree n, satisfying

$$\int_{\mathbb{C}} p_j(z)\,\overline{p_k(z)}\,\mathrm{e}^{-\frac{n}{T}Q(z)}\mathrm{d}A(z) = h_j\delta_{jk}, \qquad j,k \in \mathbb{N}\cup\{0\},$$

where $T > 0$, $\mathrm{d}A$ is the two-dimensional Lebesgue measure, h_j's are normalization constants and Q the external potential satisfying a sufficient growth condition. The polynomials are related to normal random matrices and the two-dimensional Coulomb gas [1]. The droplet, which we denote by K, is the support of the equilibrium measure that minimizes the associated potential energy. [2]

We concentrate on the cases where the potential is a non-critical Hele-Shaw potential and K is simply connected. We express the asymptotics of the polynomials in terms of the geometric quantities of the droplet using the conformal map. The droplet is very robust against modification of the potential outside of K, i.e. there is a broad class of potentials which share the same droplet. Assuming the existence of uniform asymptotics, we obtain the asymptotics of the polynomials including a new formula for the subleading correction. This describes a new kind of universality law depending on the conformal map. While in leading order this is a local law, the subleading correction contains information about the global shape of the droplet.

For the proof we use that a certain Cauchy transform of the polynomial vanishes exponentially fast inside K [3] and that the norm of the weighted polynomials is peaked along ∂K, the boundary of K. We transform the solid Cauchy transform into the Cauchy transform on ∂K, which we obtain as a solution to a Riemann-Hilbert problem on the Schottky double of the complement of K.

This presentation is based on a joint work with Seung-Yeop Lee.

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Some results concerning the fundamental solution for the time-fractional telegraph equation in higher dimensions

by

M. Manuela Rodrigues

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In this talk we present some results concerning the fundamental solution (FS) of the multidimensional time-fractional telegraph equation with time-fractional derivatives in the Caputo sense. In the Fourier domain the FS is expressed in terms of a multivariate Mittag-Leffler function. The Fourier inversion leads to a representation of the FS in terms of a H-function of two variables. An explicit series representation of the FS, depending on the parity of the dimension, is presented. As an application, we study a telegraph process with Brownian time. Finally, we present some moments of integer order of the FS, and some plots of the FS for some particular values of the dimension and of the fractional parameters.

This presentation is based on a joint work with M. Ferreira and N. Vieira

The failure of Beurling type theorem for A_4^2

by

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A closed subspace I of A^2_α is called invariant if $zf \in I$ whenever $f \in I$. For any subset $E \subset A^2_\alpha$, let $[E]$ be the smallest invariant subspace containing E. In 1996 Aleman, Richter and Sundberg [1] proved the so called Beurling-type theorem for A^2_0, which states that if I is an invariant subspace of A^2_0, then $I = [I \ominus zI]$. This theorem has been extended by Shimorin [3, 4] for A^2_α with $\alpha \in (-1, 1]$. It follows from the results obtained by Hedenmalm and Zhu [2] that such a theorem fails for $\alpha > 4$. We proved that Beurling-type theorem doesn't holds for A^2_4.

Above problem is open for $\alpha \in (1, 4)$.

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Logarithmic Coefficients Estimates of Quasi-Convex Univalent Functions

by

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Let Q denote the class of *quasi-convex* functions consisting of analytic functions f in the unit disk $\Delta = \{z \in \mathbb{C} : |z| < 1\}$ with $f(0) = 0 = f'(0) - 1$ satisfying the condition $\mathrm{Re}\frac{(zf'(z))'}{g'(z)} > 0$ where g is a convex function with $g(0) = 0 = g'(0) - 1$. The logarithmic coefficients of f are defined by the formula $\log \frac{f(z)}{z} = 2\sum_{n=1}^{\infty} \gamma_n z^n$ Recently an estimation of γ_n, $n = 1, 2, 3$ for close-to-convex functions and its subclasses has been obtained. In this paper we estimate the first three logarithmic coefficients for functions belonging to the class of quasi-convex functions.

This presentation is based on a joint work with S. Sunil Varma.

A new framework for numerical analysis of nonlinear systems: the significance of the Stahl's theory and analytic continuation via Padé approximants

by

Sina Sadeghi Baghsorkhi
University of Michigan, United States

An appropriate embedding of polynomial systems of equations into the extended complex plane renders the variables as functions of a single complex variable. The relatively recent developments in the theory of approximation of multi-valued functions in the extended complex plane give rise to a new framework for numerical analysis of these systems that has certain unique features and important industrial applications. In electricity networks the states of the underlying nonlinear AC circuits can be expressed as multi-valued algebraic functions of a single complex variable. The accurate and reliable determination of these states is imperative for control and thus for efficient and stable operation of the electricity networks. The Padé approximation is a powerful tool to solve and analyze this class of problems. This is especially important since conventional numerical methods such as Newton's method that are prevalent in industry may converge to non-physical solutions or fail to converge at all.

The underlying concepts of this new framework, namely the algebraic curves, quadratic differentials and the Stahl's theory are presented along with a critical application of Padé approximants and their zero-pole distribution in the voltage collapse study of the electricity networks.

This presentation is based on a joint work with Nikolay Ikonomov and Sergey Suetin.

On a geometric property of the Euler-Mascheroni sequence

by

Luis Salinas
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In classical Analysis the Euler Mascheroni constant $\gamma = 0.5772156649\ldots$ is defied as $\gamma = \lim_{n\to\infty}(\sum_{l=1}^{n} \frac{1}{l} - \log(n))$. We define the Euler-Mascheroni sequence as:

$$\gamma_n = -\gamma - \log(n) + \sum_{l=1}^{n} \frac{1}{l}, \qquad n \in \mathbb{N}.$$

Note that $\gamma_1 = 1 - \gamma$. Numerical experiments suggest the following

Conjecture. The function $f(z) := \frac{1}{1-\gamma} \sum_{n=1}^{\infty} \gamma_n z^n$ is universally convex.

A necessary condition for this conjecture to be true is that the sequence $\{\gamma_n\}$ is *completly monotone* (Cf. [1, Eq. (10)]), a result which has been established so far. In this direction we consider the double sequence $\{\gamma_n^k\}_{n\geq 1, k\geq 0}$ of the differences of the sequence $\{\gamma_n\}$:

$$\gamma_n^0 = \gamma_n, \quad \gamma_n^k = \gamma_n^{k-1} - \gamma_{n+1}^{k-1}.$$

By induction one easily obtains a convenient expression for γ_n^k:

Theorem. *For $k, n \in \{0\} \cup \mathbb{N}$ with $n > 0$ we have:*

$$\gamma_n^k = \sum_{l=0}^{k-1} \binom{k-1}{l} \frac{(-1)^l}{l+n+1} - \sum_{l=0}^{k} (-1)^l \binom{k}{l} \log(l+n).$$

This representation allows the proof of the completely monotonicity of the Euler-Mascheroni sequence.

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This presentation is based on a joint work with Stephan Ruscheweyh and Claudio Torres.

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On a conjecture for trigonometric sums by S. Koumandos and S. Ruscheweyh

by

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S. Koumandos and S. Ruscheweyh (2007) posed the following conjecture: Let $s_n^\mu(z) = \sum_{k=0}^{n} \frac{(\mu)_k}{k!} z^k$, $0 < \mu \leq 1, |z| < 1$. Then for $\rho \in (0,1]$ and $0 < \mu \leq \mu^*(\rho)$

$$(1-z)^\rho s_n^\mu(z) \prec \left(\frac{1+z}{1-z}\right)^\rho, \quad n \in \mathbb{N}$$

where $\mu^*(\rho)$ is the unique solution of

$$\int_0^{(\rho+1)\pi} \sin(t-\rho\pi) t^{\mu-1} dt = 0$$

This conjecture was already settled for $\rho = \frac{1}{2}, \frac{1}{4}, \frac{3}{4}$ and also true for $\rho = 1$. In this work, we prove this conjecture for an open neighbourhood of $\rho = \frac{1}{3}$ and in a weaker form for $\rho = \frac{2}{3}$. This particular value of the conjecture leads to several consequences related to Cesàro polynomials and starlike functions.

This presentation is based on a joint work with A. Swaminathan and I.I.T. Roorkee.

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Critical points of finite Blaschke products, Stieltjes polynomials, and moment problems

by

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It has been proved by several authors that for $n-1$ points in the unit disc there is an (essentially unique) Blaschke product of degree n with these points as critical points. However, this problem has a number of reformulations that still allow new insights and a simple and natural proof. In particular, it is equivalent to finding certain Stieltjes and Van Vleck polynomials which in turn permits a physical interpretation as an equilibrium problem for movable unit charges in the presence of an electrical field generated by fixed charges.

A second reformulation involves a moment problem for the canonical representation of power moments on the real axis or, equivalently, the Vandermonde factorization of a Hankel matrix. These equivalences are not only of theoretical interest, but also open up new perspectives for the design of algorithms.

This presentation is based on a joint work with Elias Wegert.

Visualizing the solution of the radial Loewner equation

by

Hirokazu Shimauchi

Department of human sciences and cultural studies, Yamanashi eiwa college

The Loewner equation provides a one-parametric family of conformal maps on the unit disk $\mathbb{D}$ whose images describe a flow of an expanding simply-connected domain on the complex plane $\mathbb{C}$. The Loewner equation has been successfully used for various problems in Geometric Function Theory. It should be noted that remarkable advances in various fields, including the cerebrated Schramm-Loewner Evolution, are found recently.

One of the standard models is called the *radial case*, following Pommerenke's characterization (see e.g. [1]). A time-parametrized holomorphic function $f_t : \mathbb{D} \to \mathbb{C}$ $(t \geq 0)$ is said to be a *(radial) Loewner chain* if;

(1) f_t is injective, i.e., a conformal map on $\mathbb{D}$, for all $t \geq 0$,
(2) $f_t(0) = 0$ and $f_t'(0) = e^t$,
(3) $f_s(\mathbb{D}) \subset f_t(\mathbb{D})$ for all $t > s \geq 0$.

Further, it satisfies the *radial Loewner equation*

$$\frac{\partial f_t(z)}{\partial t} = \frac{\partial f_t(z)}{\partial z} \cdot zp(z,t) \tag{1}$$

for all $z \in \mathbb{D}$ and almost all $t \in [0,\infty)$, where the term $p(z,t)$ is called a *Herglotz function*; measurable with respect to $t \in [0,\infty)$ for all fixed $z \in \mathbb{D}$, holomorphic with respect to $z \in \mathbb{D}$ for almost all fixed $t \in [0,\infty)$ and satisfies $\operatorname{Re} p(z,t) > 0$ for all $z \in \mathbb{D}$ and almost all $t \in [0,\infty)$. On the other hand, there exists a Loewner chain f_t that satisfies (1) with a given Herglotz function p

In this talk, we will present a simple algorithm which produces a family of polynomial approximations of the Loewner chain for a given Herglotz function. Our algorithm is based on a recursive formula of the coefficients of the Maclaurin polynomial family of the Loewner chain f_t. A justification of the recursive formula and some numerical experiments will be presented.

This presentation is based on a joint work with Ikkei Hotta

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S-contours for multiple orthogonal polynomials in the hermitian plus external source random matrix model

by

Guilherme Silva
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Multiple orthogonal polynomials (MOP's) were introduced already in the works of Hermite and his student Padé, reason why they are sometimes also called Hermite-Padé polynomials. In the past twenty years MOP's have gained a lot of attention, partially due to its connection to several models of mathematical physics, such as non-intersecting paths, planar growth and random matrix theory. In contrast to the standard orthogonality, the behavior of the zeros of multiple orthogonal polynomials when the degree gets large is not completely understood, with results available only for somewhat extremal cases (as Nikishin or Angelesco systems) or under strong symmetry conditions on the orthogonality weights.

Much more recent, and somewhat better understood, is the theory of non-hermitian orthogonal polynomials, which are polynomials orthogonal over contours on the complex plane. In his seminal works in the late 1980's Herbert Stahl introduced the notion of the S-property, which turns out to completely characterize the asymptotics of a wide class of non-hermitian orthogonal polynomials. Since then Stahl's S-property has been vastly explored in the literature, with results now available for classes of orthogonal polynomials beyond the initial ones studied by Stahl.

In this talk we focus on a class of multiple orthogonal polynomials that arise in the hermitian matrix model with external source. Starting from the existence of an appropriate algebraic equation, known as the spectral curve of the matrix model, we construct a contour with the S-property for a vector energy involving three measures and interactions of both Nikishin and Angelesco types. The existence of this spectral curve, although quite natural, is highly non-trivial. Using available results in random matrix theory we are able to complete the existence of the S-contour for some classes of potentials.

This presentation is based on a joint work with Andrei Martínez-Finkelshtein.

Asymptotically optimal point configurations for Chebyshev constants

by

Brian Simanek
Baylor University

We will discuss a discrete optimization problem that is dual to the minimum energy problem in the same way that the sphere covering problem is dual to the sphere packing problem. The problem at hand concerns arranging points on a manifold in such a way that the minimum of the potential generated by their counting measure is as large as possible. Under the appropriate assumptions, we can say that as the number of points becomes large, then the asymptotically optimal configurations of points for this problems distribute themselves like the equilibrium measure for the given kernel and manifold.

Ideals with at most countable hull in certain algebras of analytic functions

by

Andrzej Sołtysiak
Adam Mickiewicz University, Poznań, Poland

Closed ideals of subalgebras of the classical disc algebra $A(\mathbb{D})$ were investigated by many authors. We present an extension of a theorem of Agrafeuil and Zarabi ([1]) (and also Faĭvyševskiĭ ([2], [3])) showing that under certain natural assumptions and modified Ditkin's condition every closed ideal with at most countable hull of a given algebra $\mathcal{B}$ is standard.

Next, using this result we describe closed ideals with at most countable hull in algebras $\mathcal{A}^{(\alpha)}(\mathbb{C}^+)$ $(\alpha > 0)$ of bounded analytic functions on the right half-plane satisfying certain conditions on the boundary.

This presentation is based on a joint work with Antoni Wawrzyńczyk.

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Strong asymptotics for Szegő polynomials for non-smooth curves

by

Nikos Stylianopoulos
University of Cyprus

Strong asymptotics for Szegő polynomials, i.e., polynomials orthonormal with respect to the arclength measure on a rectifiable Jordan curve Γ in the complex plane, have been first derived in the early 1920's by G. Szegő, under the assumption that Γ is an analytic Jordan curve.

The transition from analytic to smooth was not obvious and it took almost half a century, in the 1960's, when P.K. Suetin has been able to derive similar asymptotics for smooth Jordan curves.

The purpose of the talk is to report on some recent results on the strong asymptotics of Szegő polynomials, in cases when Γ is a non-smooth Jordan curve, in particular, piecewise analytic without cusps.

A Möbius invariant and a nearly hyperbolic distance of a punctured sphere

by

Toshiyuki Sugawa
Tohoku University, Japan

We introduce a Möbius invariant quantity $Q(A)$ of a finite subset A of the Riemann sphere $\widehat{\mathbb{C}}$ containing at least four points. This quantity is defined in terms of the (absolute) cross ratios of quadraples of distinct points in A. One of our observations is that $Q(A)$ is, in some sense, close to the systole of the punctured sphere $X = \widehat{\mathbb{C}} \setminus A$. We then construct a distance function on X which is equivalent to the hyperbolic distance on X but easier to compute. if our distance function is modelled on the quasi-hyperbolic distance, then our distance is comparable with the hyperbolic distance with bounds depending only on $Q(A)$.

This presentation is based on a joint work with Matti Vuorinen and Tanran Zhang.

Numerical Computation of Fractional Second-Order Sturm-Liouville Problems

by

Muhammed I. Syam
UAE University, United Arab Emirates

This article is devoted to both theoretical and numerical study of the eigenvalues of nonsingular fractional second-order Sturm-Liouville problem. The fractional derivative in this paper is in the conformable fractional derivative sense. In this paper, we implement the reproducing kernel Hilbert space method to approximate the eigenvalues. Theoretical results for the considered problem are provided and proved. The main properties of the Sturm-Liouville problem are investigated. Numerical results demonstrate the accuracy of the present algorithm. Comparisons with other methods are presented.

On Julia-Carathéodory Theorem for functions with fixed initial coefficients

by

Barbara Śmiarowska
University of Warmia and Mazury in Olsztyn, Poland

The famous Julia-Carathéodory Theorem says that every analytic self-mapping of the the unit disk $\mathbf{D} := \{z \in \mathbf{C} : |z| < 1\}$ having the angular limit 1 at 1 has the angular derivative at 1 which is either a positive real number or infinity. In this talk we present a simple generalization the so-called General Boundary Lemma due to Osserman [8, p. 3515] giving sharp lower bound of the angular derivative of self-mappings of $\mathbf{D}$ depending on their initial coefficients. Examples of analytic self-mappings of $\mathbf{D}$ complete the result.

This presentation is based on a joint work with Adam Lecko.

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The Backward Taylor Shift on Bergman Spaces

by

Maike Thelen
University of Trier

In this talk, we introduce the backward Taylor shift operator T on the Bergman spaces $A^p(\Omega)$ where Ω is an open set in the complex plane and $1 \leq p < \infty$. Depending on the values of p and the complement of Ω, the linear operator T will be studied for its dynamical behaviour. In particular, we want to find situations in which the backward shift is frequently hypercyclic or mixing. In order to do so, we apply results on the approximation by Cauchy integrals for Bergman spaces according to [1].

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Meromorphic functions that share four or five pairs of values

by

Kazuya Tohge
Kanazawa University

One of the most familiar results in classical complex analysis may be Nevanlinna's four and five value theorems on meromorphic functions in the complex plane. For such functions can be expressed by means of the zero and pole sequences together with a non-vanishing entire function e^{α} where α is entire. Then the uniqueness problem with sharing values is reduced to a question of determining this exponential function e^{α} by further non-zero finite sharing values. The pair of those functions that share *four* values counting with multiplicities CM (resp. *five* values ignoring multiplicities IM) must be Möbius transformations of each other (resp. must be identical). Examples show that each of these assumptions is best possible, but there are some extensions of them by considering a mixture of the assumptions on multiplicities. It is still open whether two non-constant meromorphic functions that share *three* values IM and share *one* other value CM necessarily share all *four* values CM. This was asked by G. G. Gundersen in 1983 when he proved this is the case with *two* values IM and the other *two* values CM.

Now we consider non-constant meromorphic functions f and g sharing a *pair of values* instead of a single value. Corresponding to the *five* value theorem, T. P. Czubiak and Gundersen proved in 1997 that if f and g share *six* pairs of values IM, then f is a Möbius transformation of g. This talk is to present that two non-constant meromorphic functions f and g that share *two* pairs of values CM and share *three* other pairs of values IM are Möbius transformations of each other, that is, all *five* pairs of values CM: '2CM+3IM=5CM'. Examples of meromorphic functions that share four or five pairs of values are discussed in order to show that this result is sharp in two senses that '1CM+4CM' does not imply 5CM and '2CM+2IM' does not imply 4CM, either. Note that all these examples possess such a form as $R(e^{\alpha})$ with a rational function R. We prove our result by contradiction, that is, by making the additional assumption that f is not a Möbius transformation of g, and then arrive at a contradiction. Instead of the difference

$f - g$ when f and g share single values, we will observe two auxiliary polynomials in f and g, both of which vanish whenever the pair f, g attain the pairs of finite values. This will give an irreducible polynomial $P(x, y)$ in two variables such that $P\big(f(z), g(z)\big) \equiv 0$ under some normalizations. Picard's theorem on algebraic curves shows that f and g should be 'essentially' of the form $R(e^{\alpha})$ then. A contradiction is deduced by a more analysis of those functions $R(e^{\alpha})$ with sharp estimates on value distribution of f and g that we observe.

This presentation is based on a joint work with Gary G. Gundersen and Norbert Steinmetz.

Extremal problems of some family of holomorphic functions of several complex variables

by

EDYTA TRYBUCKA
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We introduce a new family of holomorphic functions of several complex variables in complete bounded n-circular domains of the space $\mathbf{C}^n$. We describe relation between this family and some families defined by Bavrin (see [1]) and considered by many mathematicians (for example [2], [4], [5], [6]). Moreover, we give a distortion type theorem and a sufficient condition for functions belonging to the investigated family.

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Non-real zeroes of homogeneous differential polynomials

by

Mikhail Tyaglov
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Given a real polynomial $p(z)$ with only real zeroes, we estimate the number of non-real zeroes of the differential polynomial

$$F_{\varkappa}[p](z) = p(z)p''(z) - \varkappa[p'(z)]^2,$$

where $\varkappa$ is a real number.

A counterexample to a conjecture by B. Shapiro on the number of real zeroes of the polynomial $F_{\frac{n-1}{n}}[p](z)$ in the case when the real polynomial $p(z)$ of degree n has non-real zeroes is constructed.

This presentation is based on a joint work with Mohamed Atia.

Region of Variability for a Subclass of Analytic Univalent Functions

by

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In this paper we consider the class $\mathcal{P}(\alpha),\ 0 \leq \alpha \leq 1$ consisting of non-vanishing analytic functions f in the unit disc Δ with $f(0) = 1$ and satisfying the analytic criteria

$$Re\left(\frac{zf'(z)+\alpha z^2 f''(z)}{(1-\alpha)f(z)+\alpha zf'(z)}\right) > -1,\ z \in \Delta.$$

For $z_0 \in \Delta$ and $|\lambda| \leq 1$, we determine explicitly the region of variability

$$V(z_0, \alpha, \lambda) = \{\log((1-\alpha)f(z_0) + \alpha z_0 f'(z_0)) : f \in \mathcal{P}(\alpha), f'(0) = 2\lambda(1-\alpha)\}.$$

This presentation is based on a joint work with Thomas Rosy.

Bergman-Weil expansion for holomorphic functions

by

Alekos Vidras
University of Cyprus

Let $U \subset \mathbf{C}$ be a bounded domain with piecewise smooth boundary. Motivated by ideas coming from the multiplicative theory of residue currents in the non complete intersection case as well as from weighted Cauchy or Cauchy-Pompeiu formulaes, we state division interpolation formulaes of the Lagrange type with respect to the powers of an ideal $(f_1, ..., f_m)$ (when $m > 1$) and derive from them a "balanced" Bergman-Weil type convergent expansion of h in the domain $\{z \in U\,;\, \|f(z)\| < \min_{\partial U} \|f(\zeta)\|\}$ in terms of $f^{\underline{k}}$, $\underline{k} \in \mathbf{N}^m$.

This presentation is based on a joint work with A.Yger

Fundamental solution of the time-fractional telegraph Dirac operator

by

N. Vieira
CIDMA - Center for Research and Development in Mathematics and Applications, Department of Mathematics, University of Aveiro, Portugal

In this talk we present the fundamental solution of the multidimensional time-fractional telegraph Dirac operator where the two time-fractional derivatives of orders $\alpha \in]0, 1]$ and $\beta \in]1, 2]$ are in the Caputo sense. Explicit integral and series representation of the fundamental solution are obtained for any dimension. We remark that the series representation depends on the parity of the space dimension. We present and discuss some plots of the fundamental solution for some particular values of the dimension and of the fractional parameters α and β. Finally, using the fundamental solution we study some Poisson and Cauchy problems.

This presentation is based on a joint work with M. Ferreira and M.M. Rodrigues.

Variable density node distribution: Riesz minimizers and irrational lattices

by

O. V. Vlasiuk
Vanderbilt University

We discuss two methods for distributing nodes according to a prescribed density in such a way that certain local properties are satisfied, and the resulting configuration is well-behaved. The local criteria we are interested in are minimal separation, distribution of distances to the (k-) nearest neighbors, and covering. The importance of constructing such discrete configurations is primarily due to applications to solving PDEs using radial basis functions. The second technique we employ has connections to quasi-Monte Carlo methods.

This presentation is based on a joint work with D. P. Hardin, E. B. Saff, and T. Michaels, N. Flyer, and B. Fornberg.

Refinements of discrete Hilbert-type inequalities

by

PREDRAG VUKOVIĆ
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After its discovery, the Hilbert inequality was studied by numerous authors, who either reproved it using various techniques, or applied and generalized it in many different ways. Such generalizations included inequalities with more general kernels, weight functions and integration sets, extension to a multidimensional case, and so forth. The resulting relations are usually referred to as the Hilbert-type inequalities. On the other hand, Hardy, Littlewood and Polya, noted that to every Hilbert-type inequality one can assign its equivalent form, in the sense that one implies another and vice versa. Such forms are usually called Hardy-Hilbert-type inequalities, since they are closely connected with another famous classical inequality, that is, the Hardy inequality. For the reader's convenience, Hilbert-type and Hardy-Hilbert-type inequalities will sometimes simply be referred to as the Hilbert-type inequalities. Nowadays, more than a century after its discovery, this problem area offers diverse possibilities for generalizations and extensions.

In this article we obtain a refinement of Hilbert-type inequality in discrete case. We derive a pair of equivalent inequalities, and also establish some applications in particular settings.

Metric and quasiconformal mappings

by

Matti Vuorinen
University of Turku- Finland

This talk gives an overview of my recent research interests, connected with the theory of quasiconformal (qc) and quasiregular (qr) mappings in the Euclidean space R^n, $n \geq 2$. These mappings generalize conformal maps and analytic functions to the higher dimensional case. Their theory started fifty years ago due to the work of F. W. Gehring, J. Väisälä, Yu. G. Reshetnyak, O. Martio, S. Rickman and has been further developed by many authors. When the important parameter K, the maximal dilatation of a mapping, tends to unity, we get these classical maps, conformal maps and analytic functions as the limiting case $K = 1$. The talk will discuss the distortion theory of these mappings, i.e. how qc and qr maps transform distances between points. Some novel metrics are used in this research. The talk is based on joint work with several coauthors, mostly with my former students. In particular, the three latest coauthors are G. Wang, X. Zhang, and P. Hariri.

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http://dx.doi.org/10.1090/proc/13188, arXiv:1505.00607 [math.CA].

Weighted Riesz potential theory and convergence of empirical measures

by

F. Wielonsky
Aix-Marseille University

We will first discuss classical notions from weighted Riesz potential theory in $\mathbb{R}^d$, in particular an energy minimization problem and its corresponding discretization. We then describe Bernstein-type and Bernstein-Markov estimates for "polynomial-like" functions arising from the discretization process. Finally, we apply these estimates to derive convergence of empirical measures supported on a compact subset of $\mathbb{R}^d$ as well as a large deviation principle.

This presentation is based on a joint work with T. Bloom and N. Levenberg.

Bergman kernels and domains of holomorphy

by

Paweł M. Wójcicki
Faculty of Mathematics and Information Science, Warsaw University of Technology, Poland

The Bergman kernel has become a very important tool in geometric function theory, both in one and several complex variables. In this talk I will present a well known connection between a Bergman kernel and the so called domains of holomorphy, and my own remarks on this connection.

Key words and phrases: Weighted Bergman kernel, Admissible weight, Sequence of domains.

Distance from Bloch-Type Functions to $\mathcal{Q}_K$-Type Spaces

by

Hasi Wulan
Shantou University

In 1974, Anderson, Clunie and Pommerenke asked a question: what is the closure of the bounded functions in the Bloch space? Or what is the distance from a Bloch function to H^∞, the set of bounded analytic functions. The problem is still open. In this paper, we establish a distance formula from a Bloch-type function to $Qk(p,q)$ spaces, which generalizes some known results.

Zeros of Real Random Polynomials Spanned by OPUC

by

Aaron M. Yeager
Oklahoma State University

Let $\{\varphi_i\}_{i=0}^{\infty}$ be a sequence of orthonormal polynomials on the unit circle with respect to a probability measure μ. We study zero distribution of random linear combinations of the form

$$P_n(z) = \sum_{i=0}^{n-1} \eta_i \varphi_i(z),$$

where $\eta_0, \ldots, \eta_{n-1}$ are i.i.d. standard Gaussian variables. We use the Christoffel-Darboux formula to simplify the density functions provided by Vanderbei for the expected number real and complex of zeros of P_n. From these expressions, under the assumption that μ is in the Nevai class, we deduce the limiting value of these density functions away from the unit circle. Under mere assumption that μ is doubling on subarcs of the unit circle centered at 1 and -1, we show that the expected number of real zeros of P_n is at most

$$(2/\pi)\log n + O(1),$$

and that the asymptotic equality holds when the corresponding recurrence coefficients decay no slower than $n^{-(2+\epsilon)}$. We conclude with providing results that estimate the expected number of complex zeros of P_n in shrinking neighborhoods of compact subsets of the unit circle.

This presentation is based on a joint work with Maxim L. Yattselev.

Normality and shared values of meromorphic functions with differential polynomial

by

Wenjun Yuan
Guangzhou University

In this paper, we discuss the normality and shared values of meromorphic functions with differential polynomial. We obtain the main result: Let $\mathcal{F}$ be a family of meromorphic functions in a domain D and k, q be two positive integers. Let $P(z,w) = w^q + a_{q-1}(z)w^{q-1} + \cdots + a_1(z)w$ and $H(f, f', \ldots, f^{(k)})$ be a differential polynomial with $\frac{\Gamma}{\gamma}|_H < k+1$. If $P(z, f^{(k)}) + H(f, f', \cdots, f^{(k)}) - 1$ has at most $q(k+1)-1$ distinct zeros (ignoring multiplicity) for each function $f \in \mathcal{F}, f(z) \neq 0$, then $\mathcal{F}$ is normal in D. This result generalizes that of Chang(cf. [1]) and Yuan et al. (cf. [2]).

This presentation is based on a joint work with Lixia Cui.

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The Schwarz range domain for harmonic mappings of the unit disc with boundary normalization

by

Józef Zając
The State School of Higher Education in Chełm

The family $\mathcal{F}$ of boundary normalized harmonic mappings of the unit disc into itself is studied by the authors. An expression of the Schwarz type inequalities concerning the mappings in question is presented here. In particular, introduced by the authors, the Schwarz range domain

$$\bigcup_{F\in\mathcal{F}} \{F(z) : |z| \leq r\},$$

where $0 \leq r < 1$, is well described within the class $\mathcal{F}$. Previously obtained result says that $|F(0)| \leq \frac{2}{3}$ for $F \in \mathcal{F}$, whereas the present one is describing precisely the Schwarz range domain of $F(0)$ for $F \in \mathcal{F}$. This research has been strongly motivated by certain problems of airflow mechanics.

This presentation is based on a joint work with Dariusz Partyka.

Some open problems in analysis

by

Yu.B. Zelinskii

Institute of Mathematics Ukrainian National Academy of Science, Kyiv

The shadow problem. What is minimal number of pairwise disjoint ball with the centre on sphere S^{n-1} it is enough that any straight line, getting through the centre of the sphere, crossed at least one of these ball?

This problem was considered by G. Khudaiberganov and was mentioned in the lectures of the professor L. Aizenberg, during summer mathematical schools in Kaciveli (Crimea).

In our talk we consider some related problems:

1. Let centers of balls are free (not on fixed sphere)
2. Ray convexity
3. Family of convex sets with non empty interior
4. Complex and hypercomplex spaces
5. Shadow for every points of ball
6. Tangent to sphere bundles of lines
7. Consider equal radiuses of balls
8. Change transformation group
9. Not exclude pairwise intersecting family

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Minimum Riesz energy problems for vector measures associated with a condenser with intersecting plates

by

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We study a constrained Gauss variational problem with an external field relative to the α-Riesz kernel $|x-y|^{\alpha-n}$, $\alpha \in (0,n)$, for a condenser $\mathbf{A} = (A_i)_{i\in I}$ in $\mathbb{R}^n$, $n \geqslant 3$, whose oppositely charged plates intersect each other over a set of zero capacity. Conditions sufficient for the existence of minimizers are found, and their uniqueness and vague compactness are studied. Conditions obtained are shown to be sharp. Notice that the classical Gauss variational problem (without any constraint) in such a statement would be unsolvable. We also analyze continuity of the minimizers in the vague and strong topologies when the condenser and the constraint are both varied, describe the equilibrium vector potentials, and single out their characteristic properties. Our arguments are particularly based on the simultaneous use of the vague topology and a suitable semimetric structure on a set of vector measures associated with $\mathbf{A}$, and the establishment of a completeness theorem for proper semimetric spaces. The results obtained are illustrated by several examples.

This presentation is based on a joint work with P.D. Dragnev, B. Fuglede, D.P. Hardin and E.B. Saff.

Some properties of Szegö kernel

by

Tomasz Łukasz Żynda
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In this presentation I will introduce the concept of weighted Szegö kernel. I will explain, how the definition of an admissible weight in this case is different to the analogical one for Bergman kernel. Then I will say something about how to calculate weighted Szegö kernel in specific situations. In the end, I will show how weighted Szegö kernel can be used to prove general theorems of complex analysis.

Research posters

Classes of Analytic Functions based on Fractional Operators

by

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Department of Mathematics, Faculty of Science, Sokoto State University, Sokoto, Nigeria.

With the aid of Hadamard product, we make use of linear multiplier fractional differential and integral operators to define new subclasses of analytic functions in the unit disk, $|z| < 1$. Coefficient properties concerning these classes were established.

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On quasi subordination for analytic and biunivalent function class

by

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In our present investigation, subclases $\mathcal{M}_{\Sigma}^{q,\Phi}(\gamma,\lambda,\delta)$ and $\mathcal{M}_{\Sigma_{\gamma,\lambda}}^{q,\delta}(\varphi)$ of analytic bi-univalent functions are defined and established bounds for the coefficients for these subclasses. Also several related classes are considered and connections to earlier known results are made.

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A study on harmonic functions associated with Jacobi polynomials on the triangle

by

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Harmonic functions play important role in several theorical and practical applications which are encountered in mathematics and physical science. There are many research on harmonic functions on the literature. In the present paper, we derive a harmonic function by using the Jacobi polynomials on the triangle. For this purpose, we first investigate some properties of homogeneous operators associated with Jacobi polynomials of two variables on the triangle and then by applying the Laplace operator to the Jacobi polynomials of two variables by means of these properties, we obtain a harmonic function.

This presentation is based on joint work with Fatma Tasdelen

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On the coefficient problems for a new subclass of bi-univalent functions

by

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In this work, we introduce and investigate an interesting subclass $S_{\Sigma}^{*}(\alpha,\beta)$ of analytic and bi-univalent functions in the open unit disc U. For functions belonging to the class $S_{\Sigma}^{*}(\alpha,\beta)$, we obtain estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$. Also, this work focuses on attaining upper bounds for the functional $\left|a_2a_4-a_3^2\right|$ for functions $f(z)=z+\sum\limits_{n=2}^{\infty}a_nz^n$, when it belongs to the class $S_{\Sigma}^{*}(\alpha,\beta)$.

This presentation is based on a joint work with Sibel Yalçın

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Complete monotonicity of ratios of products of entire functions

by

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In their recent paper [1], Karp and Prilepkina investigate conditions for logarithmic complete monotonicity of ratios of products of weighted gamma functions on $(0, +\infty)$, i.e. products of the form $\frac{\prod_j \Gamma(A_j x + a_j)}{\prod_j \Gamma(B_j x + b_j)}$ where the argument of each gamma function has different scaling factor. The proof there is based on the classical integral representations and asymptotic formulas of the gamma and digamma function. Noting that the reciprocal of Γ is an entire function of order 1 with negative zeros, we show that an analogue of their result holds for more general entire functions of finite order with negative zeros.

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Spectral Analysis of a self-adjoint difference equation in quantum calculus

by

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In this study, we present a polynomial-type Jost solution of a self-adjoint second order q-difference equation with matrix coefficient. By using asymptotic behavior and analytical properties of the Jost solution, we investigate the spectral properties of the operator L generated by this q-difference expression. Also, we present a condition which guarantees that the operator L has a finite number of simple eigenvalues.

This presentation is based on a joint work with Şeyhmus Yardimci

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q-Properties of Close-to-convex Functions

by

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A property of any class of analytic functions by giving the quantum calculus methods will be called q-property of this functions. The purpose of this study, which main idea is based on R.J. Libera [3], is to give q-properties of close-to-convex functions.

This presentation is based on a joint work with Yaşar Polatoğlu and Oya Mert

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Approximation of periodic functions of hight smoothness by rectangle Fourier sums

by

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We obtain asymptotic equalities for upper bounds of the deviations of the rectangle Fourier sums taken over classes of periodical functions of two variables of hight smoothness. These equalities, in corresponding cases, guarantee the solvability of the Kolmogorov–Hikol'sky problem for the rectangle Fourier sums on the specified classes of functions.

Our main result is in the following theorem (see definitions in [1,2])

Theorem 1. *Suppose that* $\psi_i(k) \in D_{q_i}$, $q_i \in (0;1)$, $\Psi_i(k) \in D_{Q_i}$, $Q_i \in (0;1)$, β_i, $\beta_i^* \in R$, $i = 1,2$. *Then the following relations hold as* $n_i \to \infty$, $i = 1, 2$

$$\mathcal{E}(C^{2\psi}_{\beta,\infty}; S_{\vec{n}}) = \sup_{f \in C^{2\psi}_{\beta,\infty}} ||f(\vec{x}) - S_{\vec{n}}(f;\vec{x})||_C = \frac{8}{\pi^2} \sum_{i=1,2} \psi_i(n_i) K(q_i) +$$

$$+ O(1) \left[\sum_{i=1,2} \frac{\psi_i(n_i) q_i}{(1-q_i) n_i} + \sum_{i=1,2} \frac{\psi_i(n_i) \varepsilon_{n_i}(\psi_i)}{(1-q_i)^2} + \Pi^{Q_1,Q_2}_{n_1,n_2}(\Psi_1,\Psi_2) \right],$$

where $K(q)$ *is the total elliptic integral of the first kind,*

$$\varepsilon_m(\psi) = \sup_{k \geq m} \left| \frac{\psi(k+1)}{\psi(k)} - q \right|, \quad q = \lim_{k\to\infty} \frac{\psi(k+1)}{\psi(k)},$$

$$\Pi^{Q_1,Q_2}_{n_1,n_2}(\Psi_1,\Psi_2) = \prod_{i=1,2} \frac{\Psi_i(n_i)}{1-Q_i} \left(1 + \frac{\varepsilon_{n_1}(\Psi_1)}{1-Q_1} + \frac{\varepsilon_{n_2}(\Psi_2)}{1-Q_2} + \frac{\varepsilon_{n_1}(\Psi_1)\varepsilon_{n_2}(\Psi_2)}{(1-Q_1)(1-Q_2)} \right),$$

$O(1)$ *is a quantity uniformly bounded with respect to* n_i, q_i, Q_i, β_i, β_i^*, $i = 1,2$.

This presentation is based on a joint work with Rovenska O.

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Asymmetric truncated Toeplitz operators on finite-dimensional spaces

by

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Truncated Toeplitz operators are compressions of classical Toeplitz operators to model spaces ([7]). Their generalizations, the so-called asymmetric truncated Toeplitz operators, were recently introduced in [1] and [2, 3]. We present some known properties of asymmetric truncated Toeplitz operators. In particular, we present their characterizations in terms of matrix representations ([6]) and compare them with the characterizations of truncated Toeplitz operators ([4]).

This presentation is based on a joint work with Bartosz Łanucha

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On the Pochhammer transformation and sign-independently hyperbolic polynomials

by

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The well-known Hutchinson's theorem states that if P be a polynomial with positive coefficients, $P(x) = \sum_{k=0}^{n} a_k x^k$, and $\frac{a_{k-1}^2}{a_{k-2}a_k} \geq 4$ for $k = 2, 3, \ldots, n$, then all the zeros of P are real. We obtain sufficient conditions for a real polynomial to be a sign-independently hyperbolic polynomial or to have real separated roots in the style of Hutchinson's theorem.

For a given $h > 0$ we also consider the linear operator $\mathcal{P}_h$ on the space of complex polynomials given by the formula $\mathcal{P}_h(x^k) = x(x-h) \cdot \ldots \cdot (x-(k-1)h)$. We call $\mathcal{P}_h$ the h-Pochhammer transformation. We investigate the properties of the Pochhammer transformation, in particular, how it affects the first and last roots of a hyperbolic polynomial. We study how the coefficients in the decomposition of a polynomial with respect to the Pochhammer basis are related to its roots. We also prove an h-analogue of the famous Schur Convolution Theorem.

This presentation is based on a joint work with Anna Vishnyakova.

Invariant Families under a starlike preserving transformation

by

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We consider any holomorphic functions f defined on the unit disk D satisfying that $f(0) = 0$ and $f(z)/z \neq 0$ for z in D. We first introduce some transformations f_a of f for $a \in D$ which preserve starlikeness when f is univalent. Next, we find a family of holomorphic functions which is invariant under our transformations, and provide a geometric characterization for functions in the family. Furthermore, we present a number of sharp growth, distortion and covering theorems for the family.

This presentation is based on a joint work with Jinxi Ma and William Ma

Julia sets appear quasiconformally in the Mandelbrot set

by

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If we zoom in a certain part of the Mandelbrot set, we can see a figure $\hat{J}$ which is very similar to a certain Julia set. Furthermore, as we zoom in the middle part of $\hat{J}$, we can see a certain nested structure which is similar to the iterated preimages of $\hat{J}$ by z^2 and finally a small Mandelbrot set $\hat{M}$ appears. We explain how to formulate this phenomena and show that this actually occurs. Also we show that this kind of nested structure exists in the Julia set J_c for c in the small Mandelbrot set $\hat{M}$.

This presentation is based on a joint work with Tomoki Kawahira

On an IFS on the space of pluriregular compact subsets of $\mathbb{C}^N$

by

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Consider the space of pluriregular polynomially convex compact subsets of $\mathbb{C}^N$ with the metric defined by the means of the pluricomplex Green function (see [K]). We list some properties of the space. We consider a family of contractive similarities defined by regular polynomial mappings $\mathbb{C}^N \longrightarrow \mathbb{C}^N$ and investigate two types of attractors of this family, which are closely related to filled-in Julia sets of the polynomial mappings (autonomous and non-autonomous).

This presentation is based on a joint work with Azza Alghamdi, Maciej Klimek and Małgorzata Stawiska.

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The Khavinson conjecture

by

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Our theme deals with an extremal problem for harmonic functions in the unit ball of $\mathbb{R}^n$. We are concerned with the pointwise sharp estimates for the gradient of real valued bounded harmonic functions. One of our main result may be formulated as follows. The sharp constants in the estimates for the absolute value of the radial derivative and the modulus of the gradient of a bounded harmonic function coincide near the boundary of the unit ball. This result partially confirms a conjecture posed by D. Khavinson. The conjecture is solved for the unit ball in $\mathbb{R}^4$. We also review some related results.

Matrix representations of truncated Hankel operators

by

Małgorzata Michalska
Maria Curie-Skłodowska University

Truncated Hankel operators are compressions of classical Hankel operators to model spaces. In this presentation we describe matrix representations of truncated Hankel operators on finite-dimensional model spaces. We then show that the obtained descriptions hold also for some infinite-dimensional cases.

This presentation is based on a joint work with Bartosz Łanucha

Converses of the Edmundson-Lah-Ribarič inequality for Shannon entropy and Csiszár divergence with applications to Zipf-Mandelbrot law

by

Rozarija Mikić
University of Zagreb Faculty of Textile Technology

Some estimates for the functional based on the f-divergence and it's generalization are obtained by applying different converses of the Jensen and Edmundson- Lah-Ribarič inequalities. Various inequalities for Shannon entropy are obtained too. The results are illustrated via Zipf-Mandelbrot law.

This presentation is based on a joint work with Gilda Pečarić and Josip Pečarić.

Asymptotics for Steklov Eigenvalues on Non-Smooth Domains

by

Josiah Park
Georgia Tech, United States

We study eigenfunctions and eigenvalues of the Dirichlet-to-Neumann operator on boxes in Euclidean spaces. We consider bounds on the counting function for the Steklov spectrum on such domains.

The generating function of Cesàro sequence and its applications

by

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Let n be a non-negative integer and the sequences $a^n = (a^n_j)_{j=0}^{\infty}$ be defined by

$$a^n_j = \binom{n+j-1}{j}. \tag{2}$$

These sequences, which are known as Cesàro numbers, play an essential role in the study of Cesàro matrices, hence we bring the first four of these sequences in below:

$$\begin{array}{llllll} a^0: & 1 & 0 & 0 & 0 & \cdots, \\ a^1: & 1 & 1 & 1 & 1 & \cdots, \\ a^2: & 1 & 2 & 3 & 4 & \cdots, \\ a^3: & 1 & 3 & 6 & 10 & \cdots. \end{array} \tag{3}$$

In the next useful lemma we claim that the above sequences a^n are the coefficients of the function $(1-z)^{-n}$ or

Lemma 1. *For $|z| < 1$, the analytic function $(1-z)^{-n}$ is the generating function of Cesàro sequence*

$$(1-z)^{-n} = \sum_{j=0}^{\infty} a^n_j z^j = \sum_{j=0}^{\infty} \binom{n+j-1}{j} z^j.$$

Proof. By differentiating $n-1$ times of the identity $(1-z)^{-1} = \sum_{j=0}^{\infty} z^j$, we get the result. □

Lemma 2. *The summation of first k coefficients of the funcion $(1-z)^{-n}$ is*

$$\sum_{j=0}^{k} \binom{n+j-1}{j} = \binom{n+k}{k}.$$

Proof. The proof is obvious. □

One can note that the relation $a_j^{n+1} = \sum_{k=0}^{j} a_k^n$ is hold for the sequences in Relation 3 which is stated in the above lemma.

In the sequel other properties of the function $(1 - z)^{-n}$ and some of its applications in operator theory, to investigate the norm of well known operators such as Hilbert, Cesàro of order n, weighted mean and backward difference operators and also the evaluation of some infinite series will be presented.

This presentation is based on a joint work with A. Erami

Application of Lie Transform to Morse Oscillator

by

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In this paper, we treat the chaotic dynamics of Morse Oscillator by using Chirikov's criteria. We achieve this by performing a Lie reduction to obtain critical value ϵ_{cr}, at which system exhibit local to global chaos, on a Hamiltonian equivalent system. The results of Lie transformation analysis and Chirikov's criteria for the oscillator are compared with numerically generated Poincare Maps.

Products of Toeplitz and Hankel operators on the Bergman space in the polydisk

by

Paweł Sobolewski
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In this paper we investigate a condition for analytic square integrable functions f, g which guarantees the boundedness of products of the Toeplitz operators $T_f T_{\bar{g}}$ densely defined on the Bergman space in the polydisk. We also give a similar condition for the products of the Hankel operators $H_f H_g^*$.

Companion matrices and stability analysis

by

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Applying multistep methods to initial value problems results in difference equations. For the stability analysis it is convenient to reformulate these as a one-step recursion in higher dimension, and the evolution is described by a companion matrix whose eigenvalues coincide with the roots of characteristic polynomial of the given difference equation.

In this work we concentrate on some technical issues arising in the stability investigation of such a recursion. We consider a family of companion matrices

$$C = \begin{pmatrix} 0 & 1 \\ -c_0 & -c_1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \zeta_1 \zeta_2 & \zeta_1 + \zeta_2 \end{pmatrix} \in \mathbf{C}^{\mathbf{2}\times\mathbf{2}},$$

with complex spectrum $\{\zeta_1, \zeta_2\}$ satisfying the stability condition

$$|\zeta_1| \leq 1, \ |\zeta_2| \leq 1, \quad \text{and} \quad |\zeta_1| < 1 \text{ if } \zeta_1 = \zeta_2.$$

Since the matrices C from our family are not normal, we generally have $\|C\|_2 > 1$. Now we aim for finding a similarity transformation of C, as a function depending on ζ_1 and ζ_2, such that the transformed matrix T satisfies $\|T\|_2 \leq 1$.

We explain how to proceed for the case of dimension $n = 2$ and give an explicit answer to our question. Furthermore we give some numerical examples for dimension $n = 2$ and also $n = 3$.

This presentation is based on a joint work with Winfried Auzinger

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Approximation Theory over the Octonions by Mongenic Functions

by

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We will identify the octonions with R^8. It is know that the space of octonions is a non-associative division algebra. There are two ways that octonionic functions have been considered analytic. The older way of doing this was developed by Fueter in the the 1920's and these functions are often referred to as Fueter-regular functions. In this sense we say a function is analytic if it is a nul-solution for a Dirac type operator. This gives a Cauchy-Riemann type of definition. More recently, Gentili and Stuppa developed an interesting approach where the octonionic analytic functions have most of the properties of analytic functions in classical complex analysis. Let $e_0 = 1, e_1, e_2, \dots e_7$ be a basis for the octonions where $|e_j| = 1, j = 0, 1 \dots e_7$. From the multiplication table of the octonions it follows that if $x \in S = \{x : x = \Sigma_{j=1}^{7} x_j e_j\}$ where $|x| = 1$, then $x^2 = -1$. So, if Ω is a domain in R^8 and $f : \Omega \to R^8$, we say f is analytic if f_I is analytic on the complex line $L_I = R + RI$. In this presentation, we discuss finding best approximation to continuous octonionic functions by both of these types of analytic functions.

This presentation is based on a joint work with Sulton Catto and Alexander Kyeyfits

Spectral asymptotic of Cauchy's operator on harmonic Bergman space on a simple connected domain and logarithmic potential type operator

by

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Mapping $C : L^2(\Omega) \to L^2(\Omega)$ defined by

$$Cf(z) = -\frac{1}{\pi}\int_{\Omega}\frac{f(\xi)}{\xi - z}dA(\xi),$$

is called Cauchy's operator (transform).

If Ω is a simply connected domain in $\mathbb{C}$ with analytic boundary in papers [6],[7] was demonstrated

$$\lim_{n\to\infty}\sqrt{n}s_n(C) = \sqrt{\frac{|\partial\Omega|}{\pi}},\ \lim_{n\to\infty} ns_n(P_a^{\Omega}C) = \frac{|\partial\Omega|}{2\pi}.$$

Here $|\partial\Omega|$ denotes the lenght of the boundary $\partial\Omega$.

We find the asymptotic behaviour of the singular numbers for the Cauchy operator restricted to the harmonic subspace $L_h^2(\Omega)$, i.e. $s_n(P_h^{\Omega}C)=O(n^{-1})$, $n \to +\infty$.

Here P_h^{Ω} is the orthogonal projection from $L^2(\Omega)$ onto $L_h^2(\Omega)$. We known that P_h is an integral operator on $L^2(D)$ induced with the harmonic reproducing kernel of the disc

$$K(z,\xi) = \frac{(1-|z|^2|\xi|^2)^2}{\pi|1-\bar{z}\xi|^4} - \frac{2}{\pi}\frac{|z|^2|\xi|^2}{|1-\bar{z}\xi|^2}.$$

Moreover, we give the lower and upper asymptotic estimate bounds for $s_n(P_h^{\Omega}C)$,

$$\frac{|\partial\Omega|}{2\pi} \leq \lim_{n\to+\infty} ns_n(P_h^{\Omega}C) \leq \frac{\sqrt{|\partial\Omega|^2 + 8\|\varphi'\|_{\infty}^2}}{2\pi},$$

where $\varphi : D \to \Omega$ is a conformal mapping.

Also, we consider the product of the harmonic Bergman projection $P_h : L^2(D) \to L_h^2(D)$ and the operator of logarithmic potential type defined by $Lf(z) = -\frac{1}{2\pi}\int_D \ln|z-\xi|f(\xi)dA(\xi)$, where D is the unit disc in $\mathbb{C}$.

We describe the asymptotic behaviour of the eigenvalues of the operator $(P_hL)^*(P_hL)$. More precisely, we proved that

$$\lim_{n\to+\infty} n^2 s_n(P_hL) = \sqrt{\frac{\pi^2}{12} - \frac{1}{16}}.$$

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On coefficient problem for close-to-convex functions

by

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This paper concerns the problem of estimating of $|a_4 - a_2a_3|$, where a_k are the coefficients of a given close-to-convex function. The bounds of this expression for various classes of analytic functions have been applied in estimating of the third Hankel determinant $H_3(1)$. The results for two subclasses of the class $\mathcal{C}$ of all close-to-convex functions are sharp. This bound is equal to 2. It is conjectured that this number is also the exact bound of $|a_4 - a_2a_3|$ for whole class $\mathcal{C}$.

This presentation is based on a joint work with Katarzyna Trąbka-Więcław

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